Electromagnetics II

Unit 1：Basics of Transmission Lines

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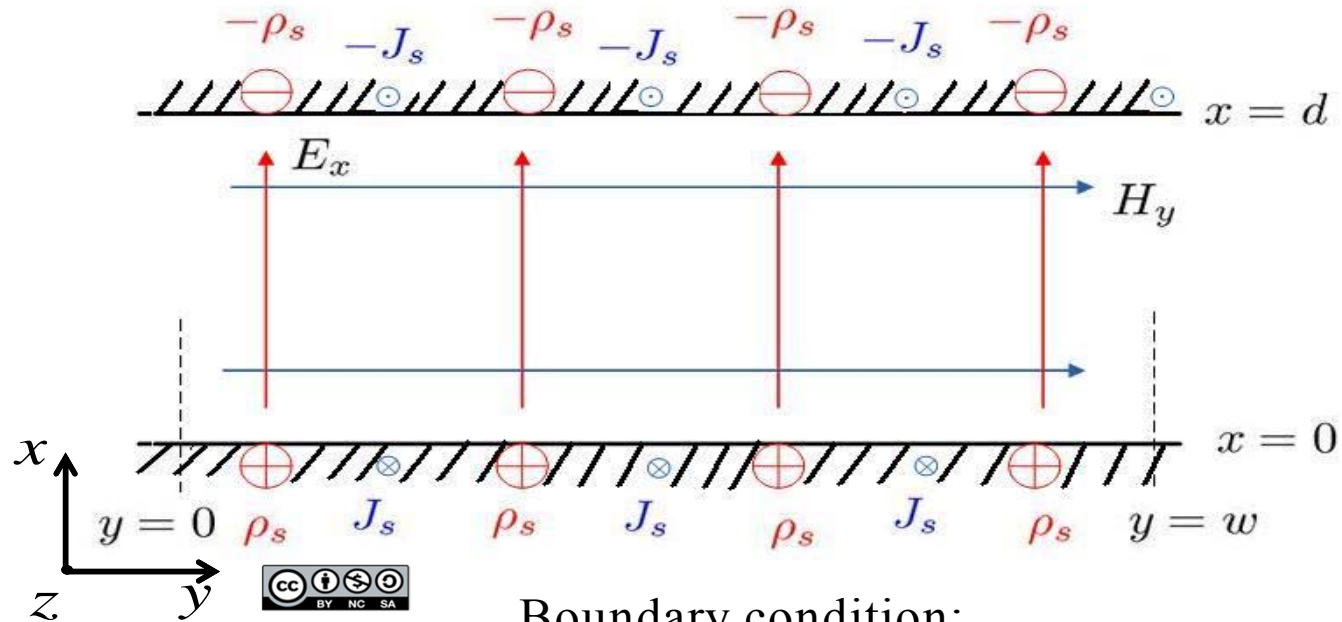
Text book: N. N. Rao, "Elements of Engineering Electromagnetics," sixth ed., Pearson, 2004.

Parallel-Plate Waveguide

Boundary condition:

$$\rho_s = \hat{n} \cdot \overline{D} = \varepsilon E_x$$

$$\overline{J}_s = \hat{n} \times \overline{H} = \hat{z} H_y$$

Maxwell's Eqn. to Wave Eqn.

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t}$$

$$\frac{\partial H_y}{\partial z} = -\varepsilon \frac{\partial E_x}{\partial t}$$

$$\Longrightarrow$$

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v_p^2} \frac{\partial^2}{\partial t^2} \right) E_x = 0$$

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v_p^2} \frac{\partial^2}{\partial t^2} \right) H_y = 0$$

Solution of Wave Equations

$$E_x\left(z,t\right) = f_+\left(t - z/v_p\right) + f_-\left(t + z/v_p\right)$$

$$H_y\left(z,t\right) = g_+\left(t - z/v_p\right) + g_-\left(t + z/v_p\right)$$

$f_+\left(t - z/v_p\right)$: propagate to $+z$ direction

$f_-\left(t + z/v_p\right)$: propagate to $-z$ direction

To maintain a fixed arguement, $\left(t - z/v_p\right)$, $z \uparrow$ as $t \uparrow$

Definition of V and I
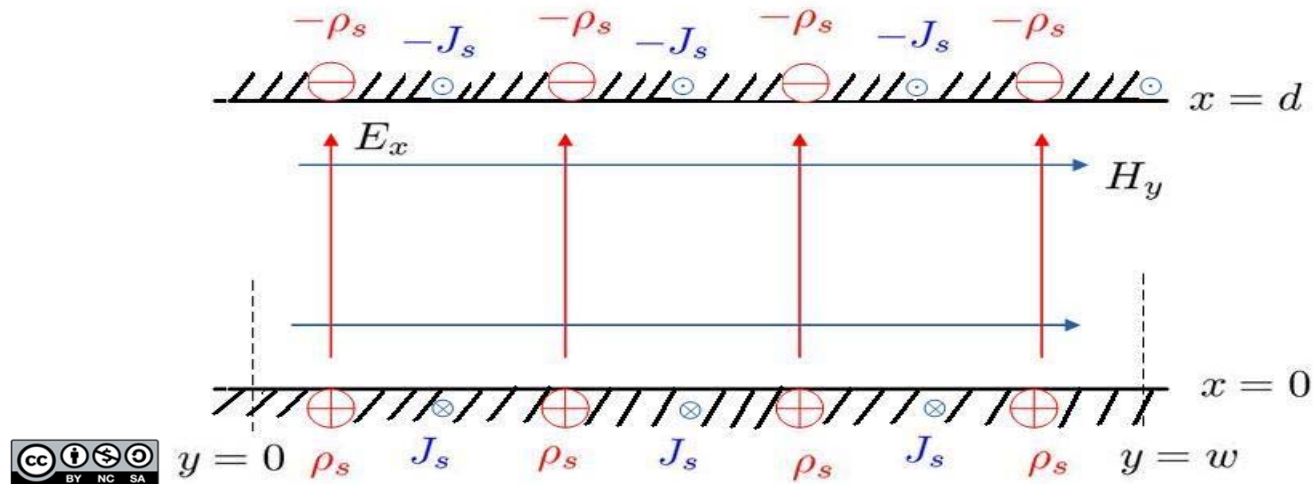
$$\int_0^d E_x(z,t)\,dx = E_x(z,t)d = V(z,t) \qquad (1)$$

$$\int_0^w H_y(z,t)\,dy = H_y(z,t)w = I(z,t) \qquad (2)$$

Telegrapher's Equations

$$\frac{\partial(E_x d)}{\partial z} = \frac{\partial V}{\partial z} = -\mu \frac{d}{w} \frac{\partial(H_y w)}{\partial t} = -L \frac{\partial(H_y w)}{\partial t} = -L \frac{\partial I}{\partial t}$$

$$\frac{\partial(H_y w)}{\partial z} = \frac{\partial I}{\partial z} = -\varepsilon \frac{w}{d} \frac{\partial(E_x d)}{\partial t} = -C \frac{\partial(E_x d)}{\partial t} = -C \frac{\partial V}{\partial t}$$

$$\Rightarrow \quad \frac{\partial V}{\partial z} = -L \frac{\partial I}{\partial t} \qquad (3)$$

$$\frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t} \qquad (4)$$

Wave Equations of *V* and *I*

$$\frac{\partial}{\partial z}(3) \Rightarrow \quad \frac{\partial^2 V}{\partial z^2} = -L\frac{\partial}{\partial t}\frac{\partial I}{\partial t} = LC\frac{\partial^2 V}{\partial t^2}$$

$$\text{where } LC = \left(\mu\,\frac{d}{w}\right)\left(\varepsilon\,\frac{w}{d}\right) = \mu\varepsilon = \frac{1}{v_p^2}$$

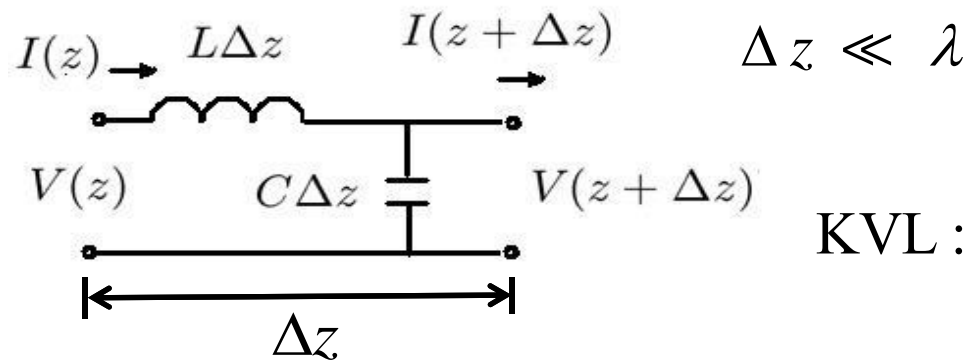$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v_p^2}\frac{\partial^2}{\partial t^2}\right)V = 0$$

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v_p^2}\frac{\partial^2}{\partial t^2}\right)I = 0$$

Solution to Wave Equations of *V*, *I*

$$V(z,t) = f_+\left(t - z/v_p\right) + f_-\left(t + z/v_p\right)$$

$$I(z,t) = g_+\left(t - z/v_p\right) + g_-\left(t + z/v_p\right)$$

Equivalent Lumped-circuit of an Infinitesimal Parallel-plate Waveguide

$$\Delta z \ll \lambda$$

$$\text{KVL}: V(z) = L\Delta z \frac{\partial I(z,t)}{\partial t} + V(z+\Delta z, t) \cong V + \frac{\partial V}{\partial z}\Delta z$$

$$\text{KCL}: I(z) = C\Delta z \frac{\partial V(z+\Delta z, t)}{\partial t} + I(z+\Delta z, t) \cong I + \frac{\partial I}{\partial z}\Delta z$$

Voltage and Current waves

$$V(z,t) = V_+\left(t - z/v_p\right) + V_-\left(t + z/v_p\right)$$

$$(3) \rightarrow \frac{\partial I}{\partial t} = -\frac{1}{L}\frac{\partial V}{\partial z} = \frac{1}{Lv_p}\left[V_+'\left(t - z/v_p\right) - V_-'\left(t + z/v_p\right)\right]$$

$$\int \frac{\partial I}{\partial t}dt \rightarrow \quad I(z,t) = \frac{1}{Z_0}\left[V_+\left(t - z/v_p\right) - V_-\left(t + z/v_p\right)\right]$$

where $\dfrac{1}{Lv_p} = \dfrac{1}{L}\sqrt{LC} = \sqrt{\dfrac{C}{L}} = \dfrac{1}{Z_0}$ Z_0: characteristic impedance (Ω)

Telegrapher's Eqn. in Frequency Domain

Time harmonic :

$$V(z,t) = \text{Re}\left\{ \underline{V}(z) e^{j\omega t} \right\}$$

$$I(z,t) = \text{Re}\left\{ \underline{I}(z) e^{j\omega t} \right\}$$

$$(3) \rightarrow \frac{\partial}{\partial z} \text{Re}\left\{ \underline{V}(z) e^{j\omega t} \right\} = -L \frac{\partial}{\partial t} \text{Re}\left\{ \underline{I}(z) e^{j\omega t} \right\}, \forall t$$

$$\Rightarrow \text{Re}\left\{ \frac{\partial \underline{V}(z)}{\partial z} e^{j\omega t} \right\} = -L \text{Re}\left\{ \underline{I}(z) \frac{\partial}{\partial t} e^{j\omega t} \right\}, \forall t$$

$$\boxed{\frac{\partial}{\partial t} \rightarrow j\omega}$$

$$\Rightarrow \boxed{\frac{\partial \underline{V}}{\partial z} = -j\omega L \underline{I} \qquad (5)} \quad \boxed{\frac{\partial \underline{I}}{\partial z} = -j\omega C \underline{V} \qquad (6)}$$

Phasor Form of *V* and *I*

solve $(5),(6)$

$$\Rightarrow \boxed{\underline{V}(z) = V_+ e^{-jkz} + V_- e^{jkz}}$$

$$\Rightarrow V(z,t) = \text{Re}\left\{\underline{V}(z) e^{j\omega t}\right\}$$

$$= |V_+| \cos(\omega t - kz + \phi_+) + |V_-| \cos(\omega t + kz + \phi_-)$$

$$(5) \rightarrow \underline{I} = -\frac{1}{j\omega L}\frac{\partial \underline{V}}{\partial z} = \frac{k}{\omega L} V_+ e^{-jkz} - \frac{k}{\omega L} V_- e^{jkz} = \frac{1}{Z_0}\left(V_+ e^{-jkz} - V_- e^{jkz}\right)$$

$$\boxed{\underline{I} = \frac{1}{Z_0}\left(V_+ e^{-jkz} - V_- e^{jkz}\right)}$$

Voltage and Current Waves along a Transmission Line
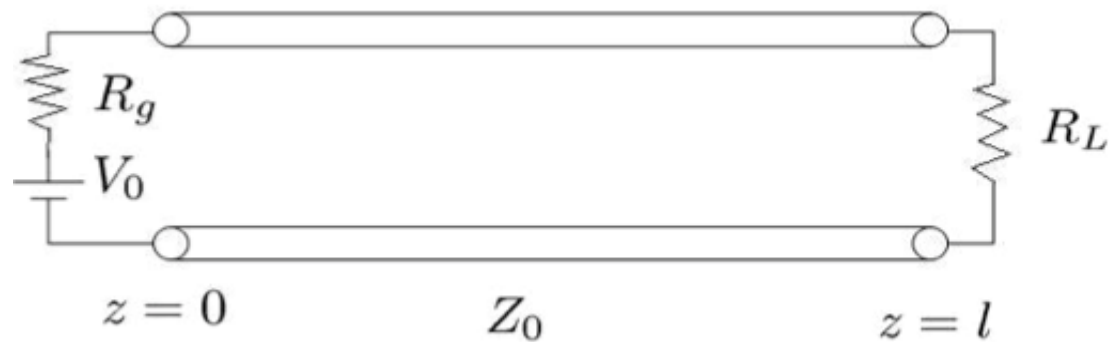
$$V(z,t) = V_+\left(t - z/v_p\right) + V_-\left(t + z/v_p\right)$$

$$I(z,t) = \frac{1}{Z_0}V_+\left(t - z/v_p\right) - \frac{1}{Z_0}V_-\left(t + z/v_p\right)$$

Voltage Reflection Coefficient by Transient-state Analysis (1)

KVL at source terminal:

$$V_+ = R_g I_+ + V_+ = R_g \frac{V_+}{Z_0} + V_+$$

$t = 0$

R_g

$+$

V_+

V_0

$-$
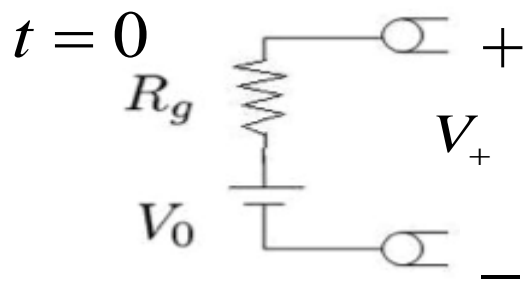
$$\Rightarrow V_+ = V_0 \frac{Z_0}{R_g + Z_0}$$

$$V_L = R_L I_L$$

KVL at load terminal:

$t = l / v_p$

$+$

$$V_L = V^+ + V^-$$

$-$
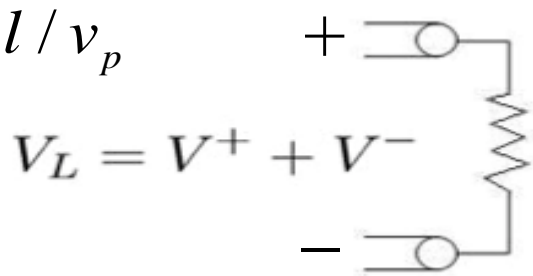
$$V_+ + V_- = R_L \left(\frac{V_+}{Z_0} - \frac{V_-}{Z_0} \right)$$

$$\Rightarrow \quad V_- = V_+ \boxed{\frac{R_L - Z_0}{R_L + Z_0} = \Gamma_v} V_+$$

Voltage Reflection Coefficient by Transient-state Analysis (2)

$$V_0 = R_g I_s + V_s = R_g \left(\frac{V_s}{Z_0} - \frac{V_-}{Z_0} + \frac{V'_+}{Z_0} \right) + V_+ + V_- + V'_+$$

$$V'_+ = V_- \boxed{\left(\frac{R_g - Z_0}{R_g + Z_0} \right) = \Gamma_s} V_-$$

$$t = 2l / v_p$$

$$V_s = V_+ + V_- + V'_+$$

R_g

V_0
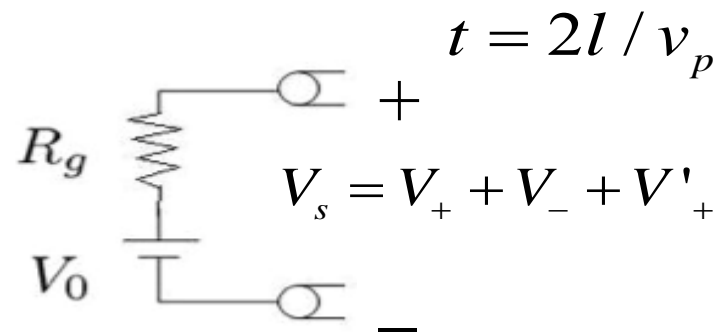
current reflection coefficient: $\Gamma_I = \dfrac{I_-}{I_+} = \dfrac{-V_- \big/ Z_0}{V_+ \big/ Z_0} = -\Gamma_v$
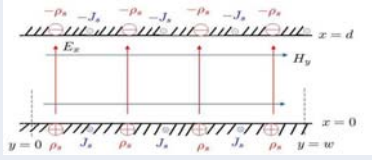
Steady-state Voltage at Source Terminal

$$t = \infty$$
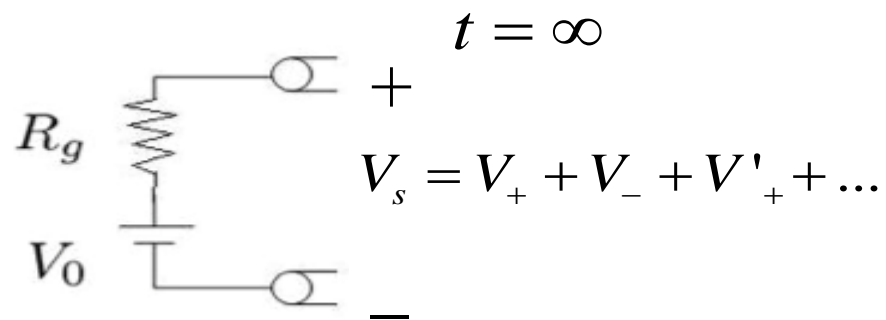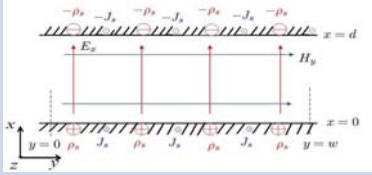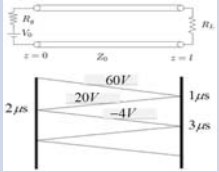
$$V_s = V_+ + V_- + V'_+ + \ldots$$

$$V(z = 0, t = \infty) = V_s = V_+ + V_- + V'_+ + V'_- + \cdots$$

$$= V_+\left(1 + \Gamma_L\right)\left(1 + \Gamma_L\Gamma_s + \Gamma_L^2\Gamma_s^2 + \cdots\right)$$

$$= V_+\left(1 + \Gamma_L\right)\frac{1}{1 - \Gamma_L\Gamma_s}$$

Space-time Diagram
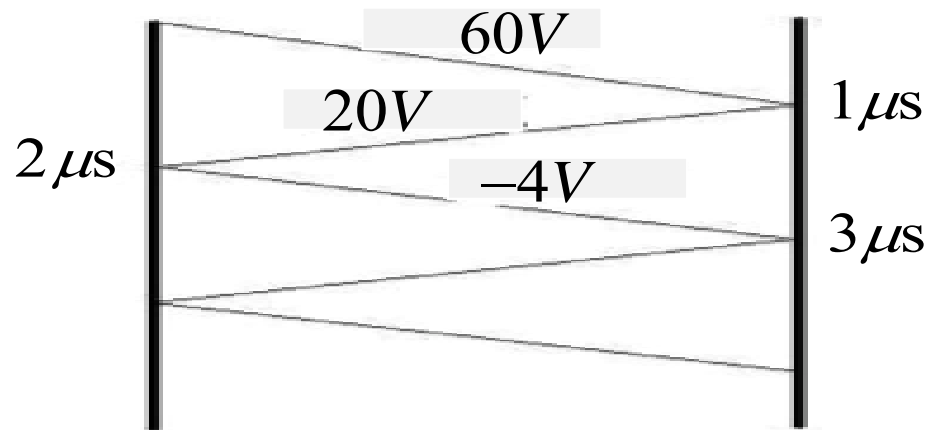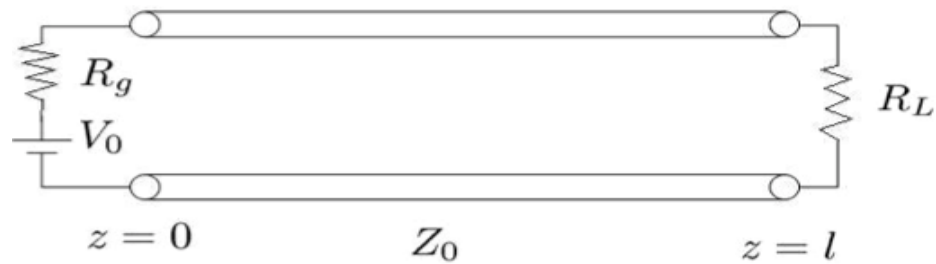
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