

Electromagnetics II

Unit 1 : Basics of Transmission Lines

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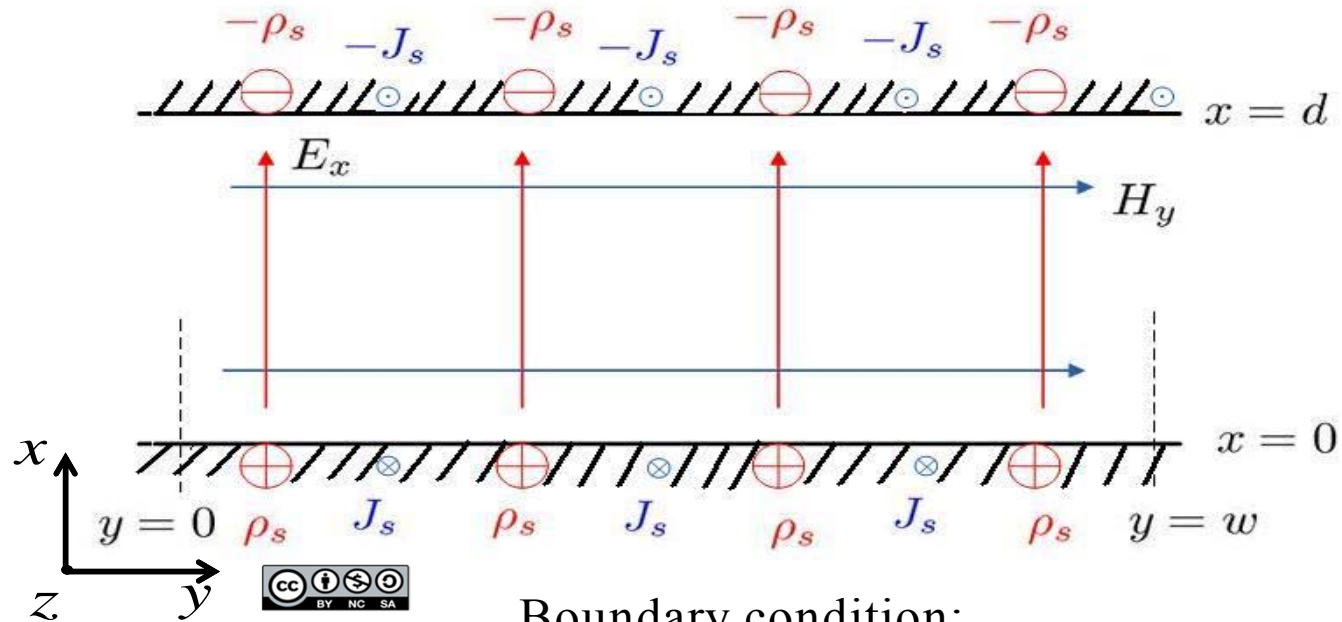
Text book: N. N. Rao, “Elements of Engineering Electromagnetics,” sixth ed., Pearson, 2004.



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Parallel-Plate Waveguide



Boundary condition:

$$\rho_s = \hat{n} \cdot \bar{D} = \epsilon E_x$$

$$\bar{J}_s = \hat{n} \times \bar{H} = \hat{z} H_y$$

Maxwell's Eqn. to Wave Eqn.

$$\begin{aligned} \frac{\partial E_x}{\partial z} &= -\mu \frac{\partial H_y}{\partial t} \\ \frac{\partial H_y}{\partial z} &= -\varepsilon \frac{\partial E_x}{\partial t} \end{aligned} \quad \Rightarrow \quad \begin{aligned} \left(\frac{\partial^2}{\partial z^2} - \frac{1}{v_p^2} \frac{\partial^2}{\partial t^2} \right) E_x &= 0 \\ \left(\frac{\partial^2}{\partial z^2} - \frac{1}{v_p^2} \frac{\partial^2}{\partial t^2} \right) H_y &= 0 \end{aligned}$$

Solution of Wave Equations

$$E_x(z,t) = f_+(t - z/v_p) + f_-(t + z/v_p)$$

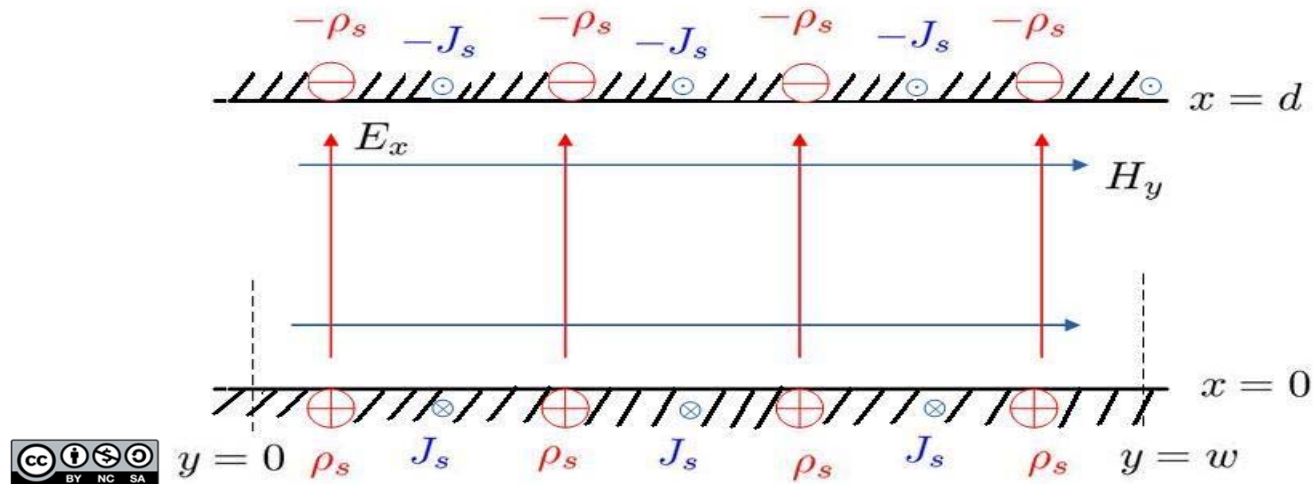
$$H_y(z,t) = g_+(t - z/v_p) + g_-(t + z/v_p)$$

$f_+(t - z/v_p)$: propagate to $+z$ direction

$f_-(t + z/v_p)$: propagate to $-z$ direction

To maintain a fixed argument, $(t - z/v_p)$, $z \uparrow$ as $t \uparrow$

Definition of V and I



$$\int_0^d E_x(z,t) dx = E_x(z,t) d = V(z,t) \quad (1)$$

$$\int_0^w H_y(z,t) dy = H_y(z,t) w = I(z,t) \quad (2)$$

Telegrapher's Equations

$$\frac{\partial(E_x d)}{\partial z} = \frac{\partial \mathcal{V}}{\partial z} = -\mu \frac{d}{w} \frac{\partial(H_y w)}{\partial t} = -L \frac{\partial(H_y w)}{\partial t} = -L \frac{\partial I}{\partial t}$$
$$\frac{\partial(H_y w)}{\partial z} = \frac{\partial I}{\partial z} = -\epsilon \frac{w}{d} \frac{\partial(E_x d)}{\partial t} = -C \frac{\partial(E_x d)}{\partial t} = -C \frac{\partial \mathcal{V}}{\partial t}$$

$$\frac{\partial \mathcal{V}}{\partial z} = -L \frac{\partial I}{\partial t} \quad (3)$$

$$\Rightarrow \frac{\partial I}{\partial z} = -C \frac{\partial \mathcal{V}}{\partial t} \quad (4)$$

Wave Equations of V and I

$$\frac{\partial}{\partial z}(3) \Rightarrow \frac{\partial^2 V}{\partial z^2} = -L \frac{\partial}{\partial t} \frac{\partial I}{\partial t} = LC \frac{\partial^2 V}{\partial t^2}$$

$$\text{where } LC = \left(\mu \frac{d}{w} \right) \left(\varepsilon \frac{w}{d} \right) = \mu \varepsilon = \frac{1}{v_p^2}$$

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v_p^2} \frac{\partial^2}{\partial t^2} \right) V = 0$$

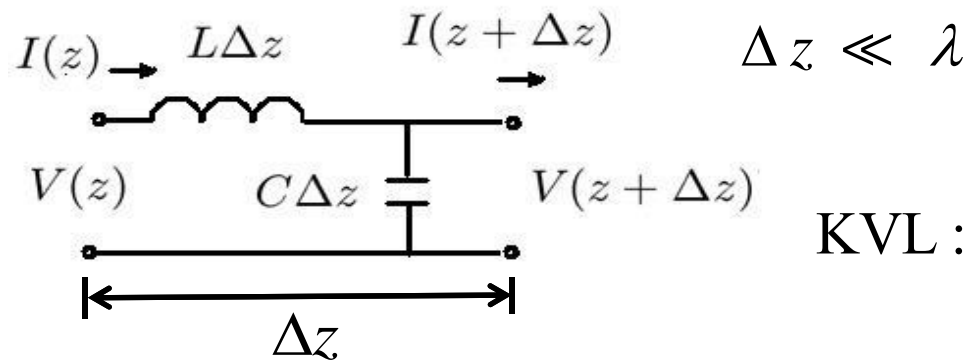
$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v_p^2} \frac{\partial^2}{\partial t^2} \right) I = 0$$

Solution to Wave Equations of V, I

$$V(z, t) = f_+ \left(t - z / v_p \right) + f_- \left(t + z / v_p \right)$$

$$I(z, t) = g_+ \left(t - z / v_p \right) + g_- \left(t + z / v_p \right)$$

Equivalent Lumped-circuit of an Infinitesimal Parallel-plate Waveguide



$$\text{KVL : } V(z) = L\Delta z \frac{\partial I(z,t)}{\partial t} + V(z + \Delta z, t) \cong V + \frac{\partial V}{\partial z} \Delta z$$

$$\text{KCL : } I(z) = C\Delta z \frac{\partial V(z + \Delta z, t)}{\partial t} + I(z + \Delta z, t) \cong I + \frac{\partial I}{\partial z} \Delta z$$

Voltage and Current waves

$$V(z, t) = V_+(t - z/v_p) + V_-(t + z/v_p)$$

$$(3) \rightarrow \frac{\partial I}{\partial t} = -\frac{1}{L} \frac{\partial V}{\partial z} = \frac{1}{Lv_p} \left[V'_+(t - z/v_p) - V'_-(t + z/v_p) \right]$$

$$\int \frac{\partial I}{\partial t} dt \rightarrow I(z, t) = \frac{1}{Z_0} \left[V_+(t - z/v_p) - V_-(t + z/v_p) \right]$$

where $\frac{1}{Lv_p} = \frac{1}{L} \sqrt{LC} = \sqrt{\frac{C}{L}} = \frac{1}{Z_0}$ Z_0 : characteristic impedance (Ω)

Telegrapher's Eqn. in Frequency Domain

Time harmonic :

$$V(z,t) = \text{Re} \{ \underline{V}(z) e^{j\omega t} \}$$

$$I(z,t) = \text{Re} \{ \underline{I}(z) e^{j\omega t} \}$$

$$(3) \rightarrow \frac{\partial}{\partial z} \text{Re} \{ \underline{V}(z) e^{j\omega t} \} = -L \frac{\partial}{\partial t} \text{Re} \{ \underline{I}(z) e^{j\omega t} \}, \forall t$$

$$\Rightarrow \text{Re} \left\{ \frac{\partial \underline{V}(z)}{\partial z} e^{j\omega t} \right\} = -L \text{Re} \left\{ \underline{I}(z) \frac{\partial}{\partial t} e^{j\omega t} \right\}, \forall t$$

$$\frac{\partial}{\partial t} \rightarrow j\omega$$

$$\Rightarrow \frac{\partial \underline{V}}{\partial z} = -j\omega L \underline{I} \quad (5) \quad \left| \quad \frac{\partial \underline{I}}{\partial z} = -j\omega C \underline{V} \quad (6)$$

Phasor Form of V and I

solve (5),(6)

$$\Rightarrow \underline{V}(z) = V_+ e^{-jkz} + V_- e^{jkz}$$

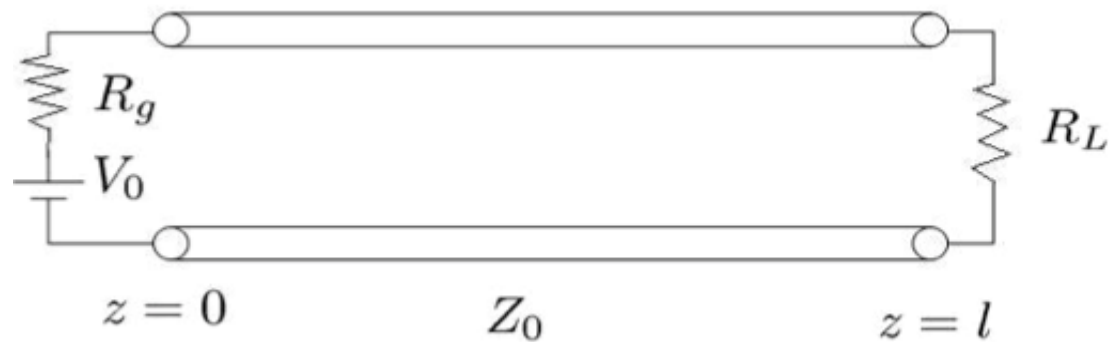
$$\Rightarrow V(z,t) = \text{Re} \{ \underline{V}(z) e^{j\omega t} \}$$

$$= |V_+| \cos(\omega t - kz + \phi_+) + |V_-| \cos(\omega t + kz + \phi_-)$$

$$(5) \rightarrow \underline{I} = -\frac{1}{j\omega L} \frac{\partial \underline{V}}{\partial z} = \frac{k}{\omega L} V_+ e^{-jkz} - \frac{k}{\omega L} V_- e^{jkz} = \frac{1}{Z_0} (V_+ e^{-jkz} - V_- e^{jkz})$$

$$\underline{I} = \frac{1}{Z_0} (V_+ e^{-jkz} - V_- e^{jkz})$$

Voltage and Current Waves along a Transmission Line

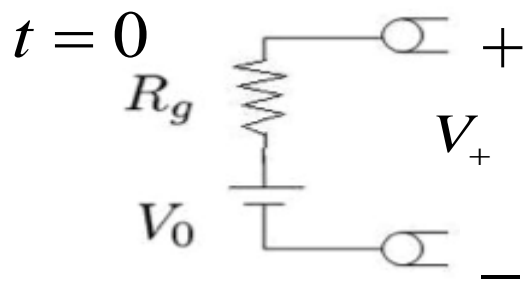


$$V(z,t) = V_+(t - z/v_p) + V_-(t + z/v_p)$$

$$I(z,t) = \frac{1}{Z_0} V_+(t - z/v_p) - \frac{1}{Z_0} V_-(t + z/v_p)$$

Voltage Reflection Coefficient by Transient-state Analysis (1)

KVL at source terminal: $V_+ = R_g I_+ + V_+ = R_g \frac{V_+}{Z_0} + V_+$

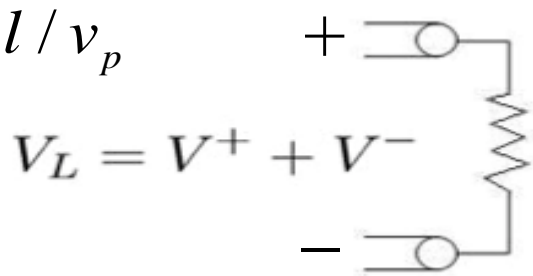


$$\Rightarrow V_+ = V_0 \frac{Z_0}{R_g + Z_0}$$

$$V_L = R_L I_L$$

KVL at load terminal:

$t = l / v_p$



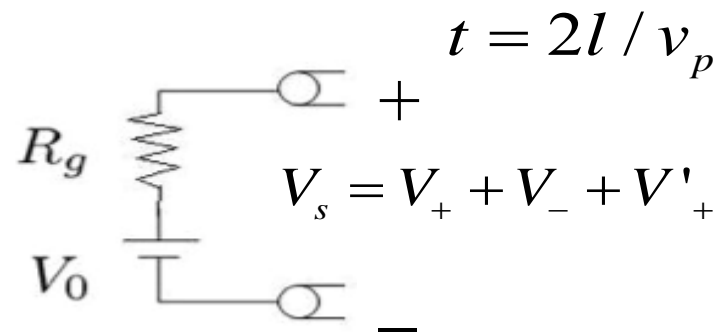
$$V_+ + V_- = R_L \left(\frac{V_+}{Z_0} - \frac{V_-}{Z_0} \right)$$

$$\Rightarrow V_- = V_+ \frac{R_L - Z_0}{R_L + Z_0} = \Gamma_v V_+$$

Voltage Reflection Coefficient by Transient-state Analysis (2)

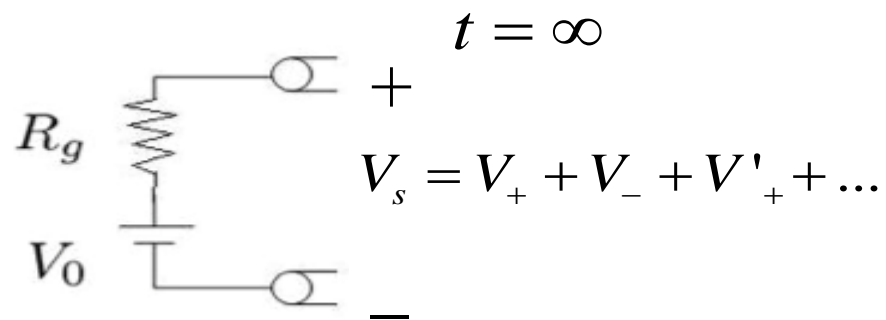
$$V_0 = R_g I_s + V_s = R_g \left(\frac{V_s}{Z_0} - \frac{V_-}{Z_0} + \frac{V'_+}{Z_0} \right) + V_+ + V_- + V'_+$$

$$V'_+ = V_- \left[\frac{R_g - Z_0}{R_g + Z_0} \right] = \Gamma_s V_-$$



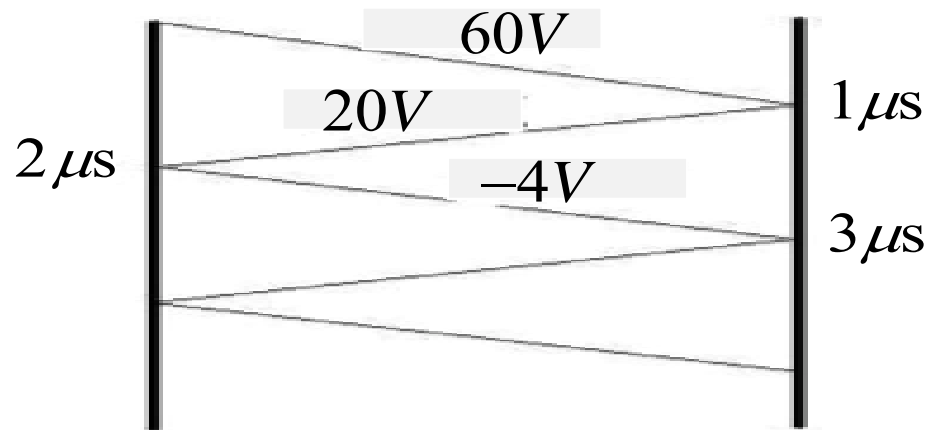
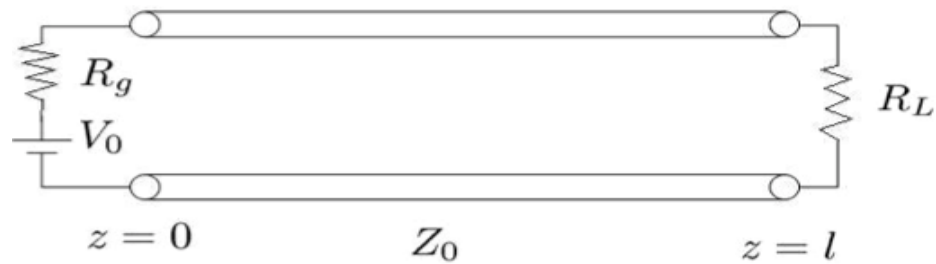
current reflection coefficient: $\Gamma_I = \frac{I_-}{I_+} = \frac{-V_- / Z_0}{V_+ / Z_0} = -\Gamma_v$

Steady-state Voltage at Source Terminal

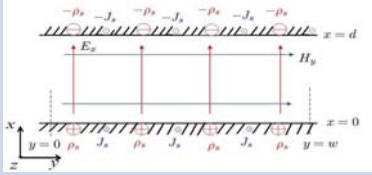

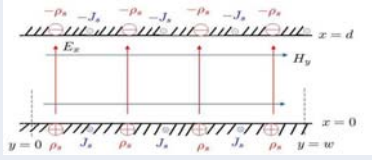

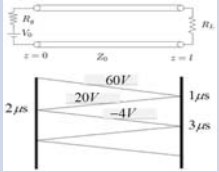



$$\begin{aligned}
 V(z=0, t=\infty) &= V_s = V_+ + V_- + V'_+ + V'_- + \dots \\
 &= V_+ (1 + \Gamma_L) (1 + \Gamma_L \Gamma_s + \Gamma_L^2 \Gamma_s^2 + \dots) \\
 &= V_+ (1 + \Gamma_L) \frac{1}{1 - \Gamma_L \Gamma_s}
 \end{aligned}$$

Space-time Diagram



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2	 <p>A diagram of a parallel plate capacitor. The top plate is at $x=d$ and the bottom plate is at $x=0$. The top plate has surface charge densities $-\rho_s$ and $-J_s$. The bottom plate has surface charge densities ρ_s and J_s. The electric field E_x is shown pointing from the bottom plate to the top plate. The magnetic field H_y is shown pointing to the right. A coordinate system with x, y, and z axes is shown at the bottom left.</p>	 <p>CC BY NC SA</p>	<p>Jean-Fu Kiang / National Taiwan University This work is licensed by Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Taiwan</p>
5	 <p>A diagram of a parallel plate capacitor, identical to the one on page 2, showing surface charge densities, electric field E_x, and magnetic field H_y between plates at $x=0$ and $x=d$.</p>	 <p>CC BY NC SA</p>	<p>Jean-Fu Kiang / National Taiwan University This work is licensed by Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Taiwan</p>
17	 <p>A diagram of a transmission line. The top part shows a transmission line with characteristic impedance Z_0 and a load impedance Z_L at $z=l$. The bottom part shows a voltage wave incident from the left. The incident wave has a peak voltage of $60V$. The reflected wave has a peak voltage of $20V$. The total voltage is $40V$. The time axis is marked with $2\mu s$, $1\mu s$, and $3\mu s$.</p>	 <p>CC BY NC SA</p>	<p>Jean-Fu Kiang / National Taiwan University This work is licensed by Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Taiwan</p>