NTU OpenCourseWare

Maxwell and His Equations

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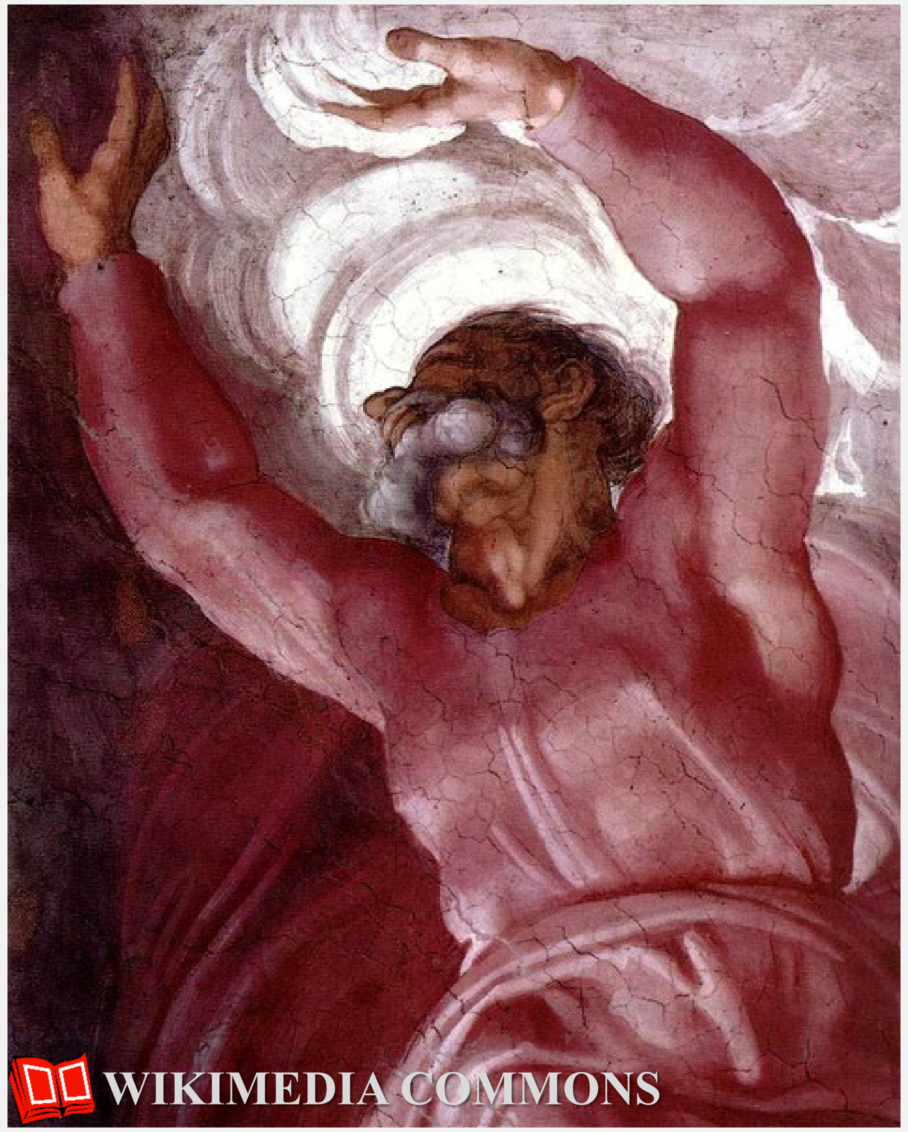
The Beginning, Genesis 1, Old Testament

1. In the beginning God created the heavens and the earth.
2. Now the earth was formless and empty, darkness was over the surface of the deep, and the Spirit of God was hovering over the waters.
3. And God said, “Let there be light,” and there was light.
4. God saw that the light was good, and he separated the light from the darkness.

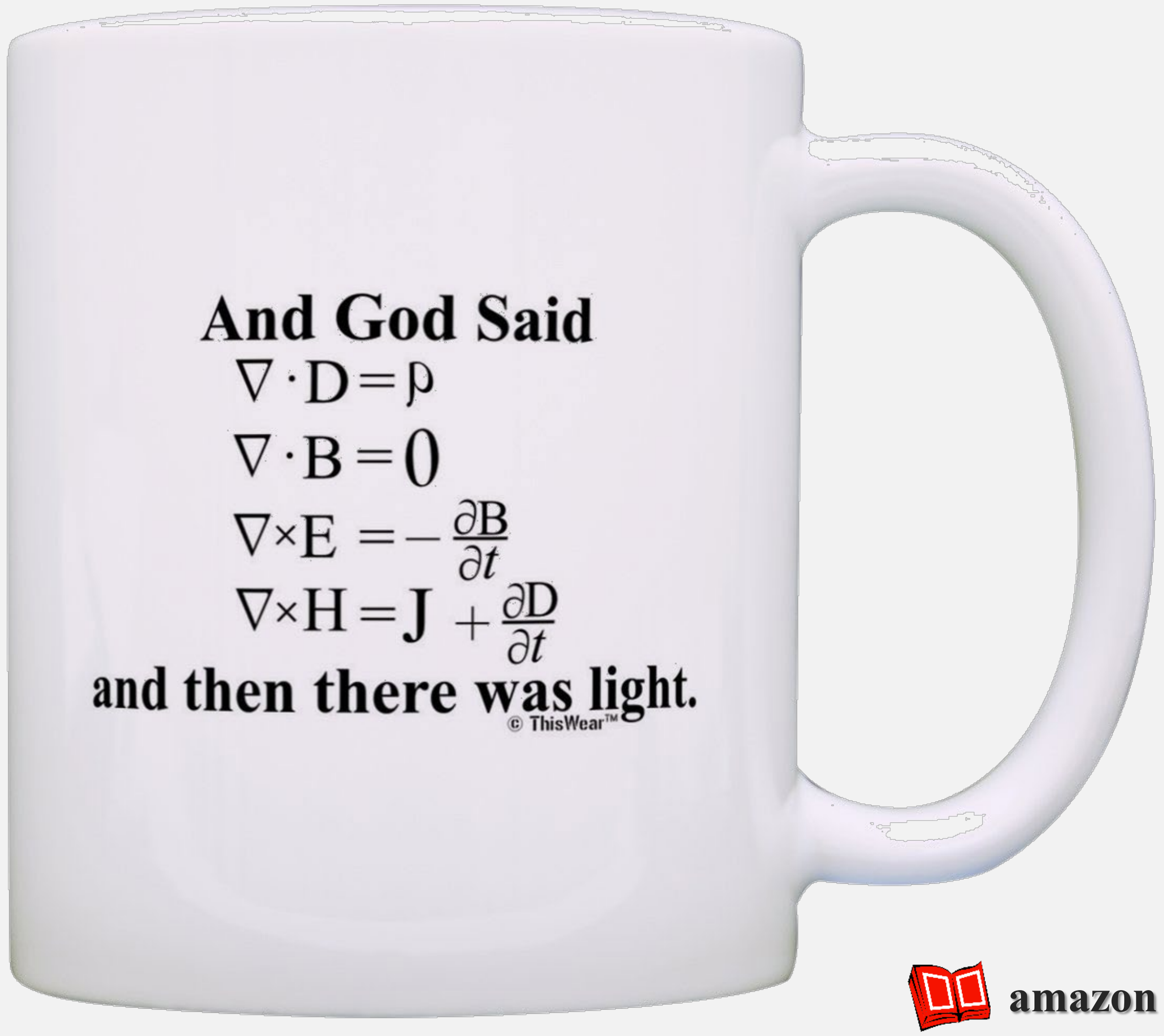


The Sistine Chapel Ceiling / Michelangelo Buonarroti

https://commons.wikimedia.org/wiki/File:Dividing_Light_from_Darkness.jpg

amazon

https://www.amazon.com/Maxwell-Equations-There-Light-Shirt/dp/B07KVSVK8G



amazon

https://www.amazon.com/Funny-Maxwells-Equations-There-Coffee/dp/B017OE2KTI/ref=sr_1_1?crid=P8CII24NYLH&keywords=maxwell%2Bequation%2Bmug&qid=1566367884&s=gateway&sprefix=Maxwell%2Beq%2Caps%2C358&sr=8-1&th=1

https://flic.kr/p/QGakzx

Inaugural Lecture of Maxwell, King’s College, 1860

In his inaugural lecture, as Professor of Physics and Astronomy at King’s College, London, he said:

In this class I hope you will learn not merely results, or formulae applicable to cases that may possibly occur in our practice afterwards, but the principles on which those formulae depend, and without which the formulae are mere mental rubbish.

Google Books
https://books.google.com.tw/books?id=IMl7AgAAQBAJ&pg=PA8&lpg=PA8&dq=in+this+class+i+hope+you+will+learn+not+merely+results+maxwell&source=bl&ots=UX09wsz5Ux&sig=ACfU3U2-n1ZHGORdlF8DB8ct9LBdYLl-CA&hl=en&sa=X&ved=2ahUKEwjDyOvg-cn1AhUyLqYKHYQgCVcQ6AF6BAgCEAM#v=onepage&q&f=false

https://commons.wikimedia.org/wiki/File:James_Clerk_Maxwell.png

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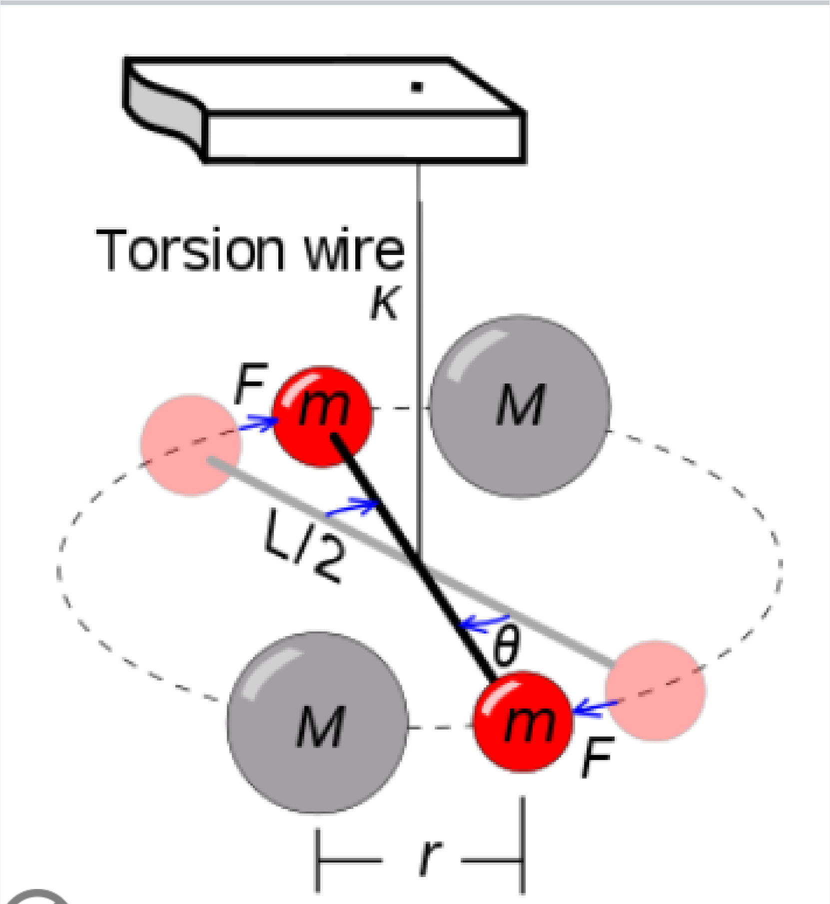
Shyh-Kang Jeng in front of King’s College, London, 2018

Henry Cavendish and Maxwell Center, Cavendish Laboratory



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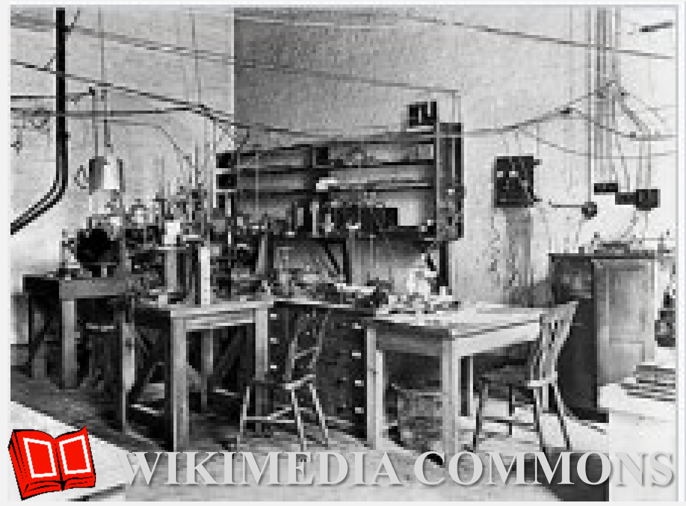
https://commons.wikimedia.org/wiki/File:Cavendish_Henry_signature.jpg



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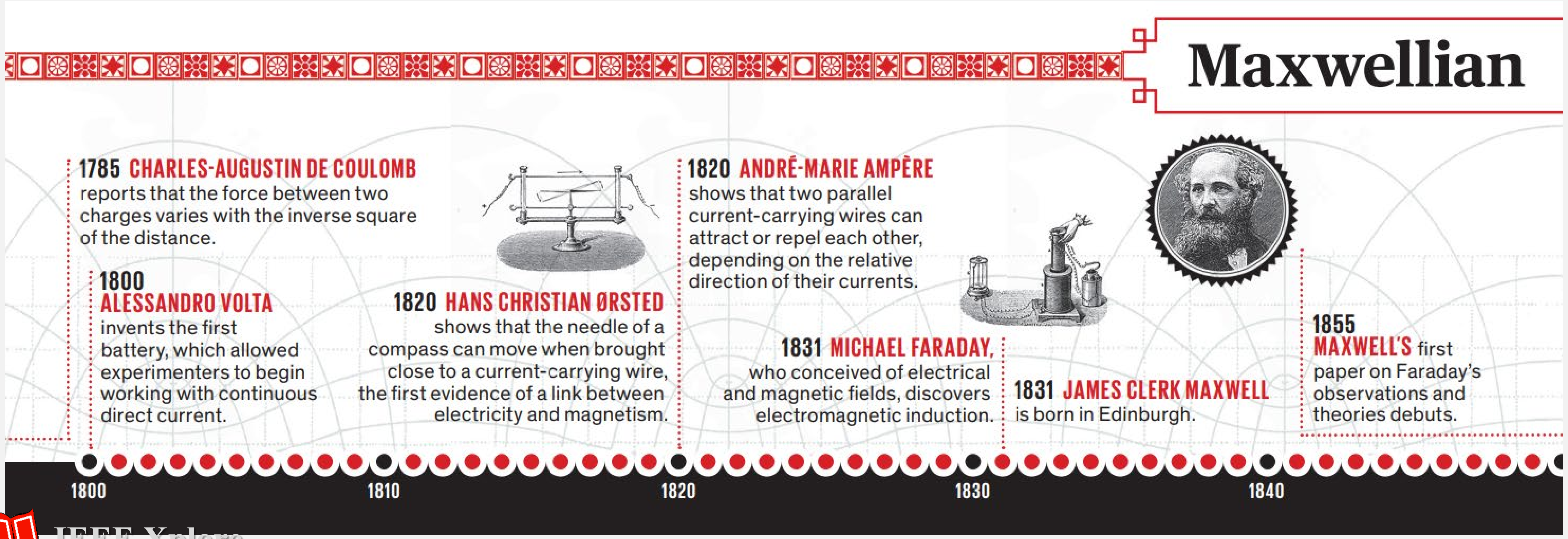
https://commons.wikimedia.org/wiki/File:Sir_Ernest_Rutherfords_laboratory,_early_20th_century._(9660575343).jpg

University of Cambridge

https://www.cam.ac.uk/research/news/pioneering-centre-for-physical-sciences-industry-collaborations-opens-at-university-of-cambridge

Long Road to Maxwell’s Equations



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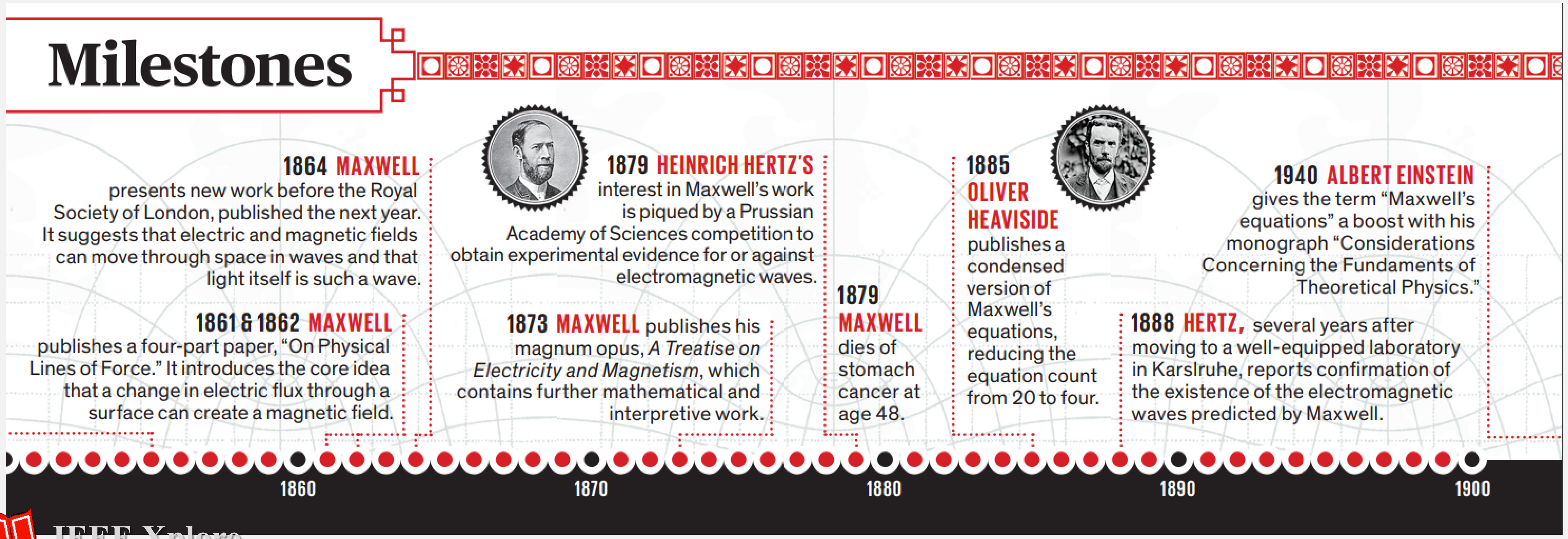

Outline

- Life of James Clark Maxwell
- Electricity and Magnetism before Maxwell
- Evolution of Maxwell’s Equations
- “Derivation” of Maxwell’s Equations

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Time Line

DATE	EVENT
1831	Born at 14 India Street, Edinburgh and grew up at Glenlair
1841 - 1847	Edinburgh Academy
1847 - 1850	Edinburgh University
1850 – 1856	Cambridge University
1856 - 1860	Marischal College, Aberdeen
1860 - 1865	King's College, London
1865 - 1871	Glenlair
1871 – 1879	Cambridge University

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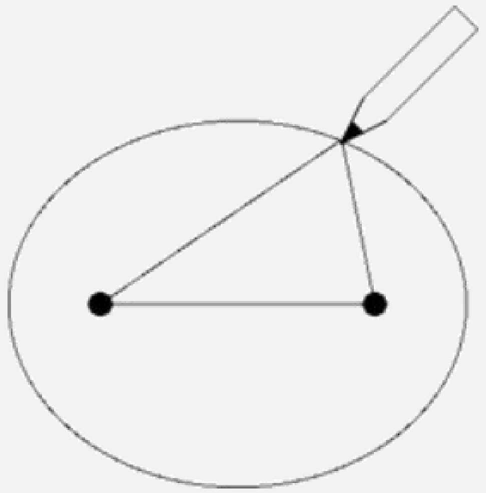
https://commons.wikimedia.org/wiki/File:James_Clerk_Maxwell.png

WIKIMEDIA COMMONS

First Paper, Drawing Oval Curves, 1846



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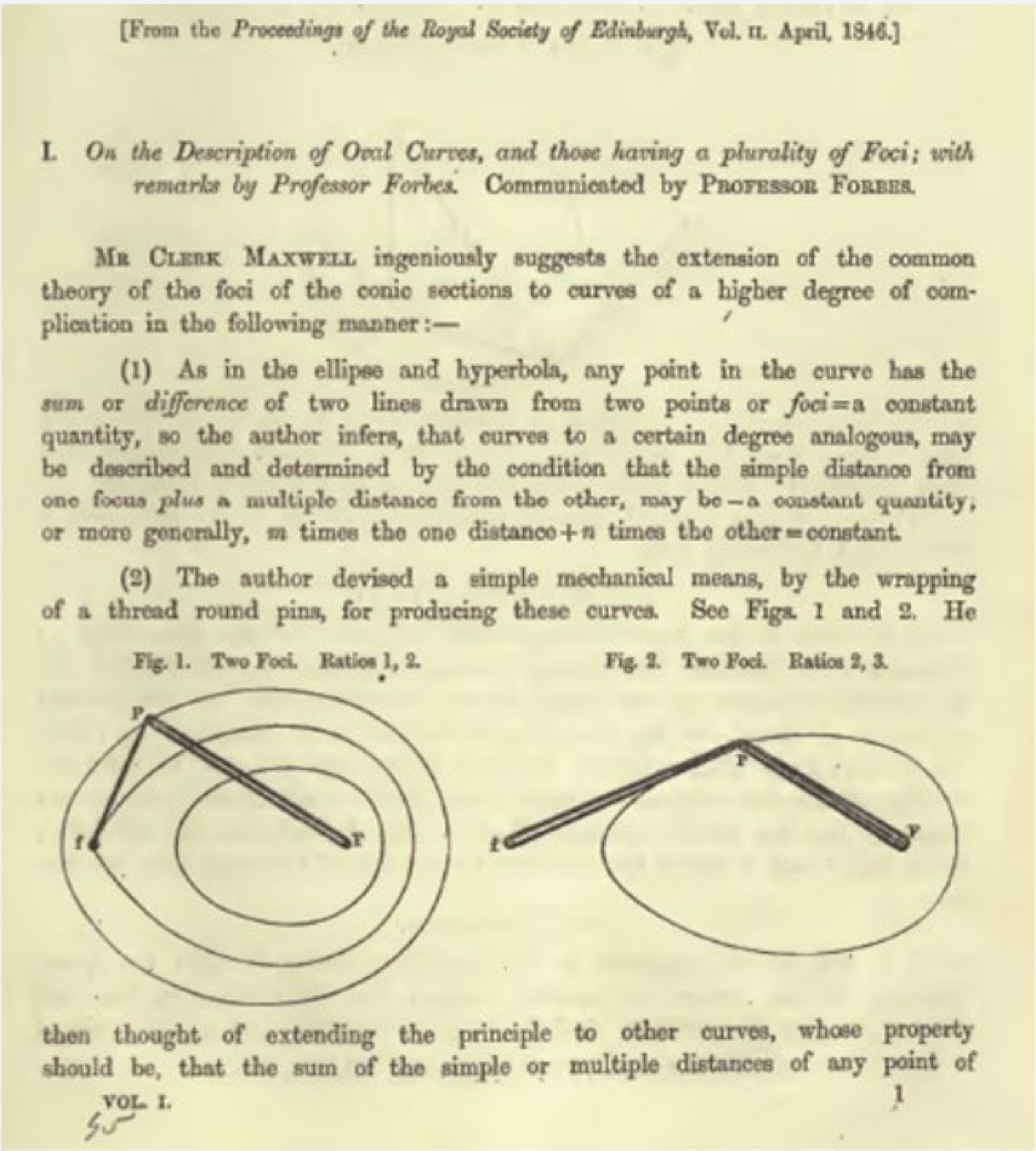

[From the *Proceedings of the Royal Society of Edinburgh*, Vol. II. April, 1846.]

I. *On the Description of Oval Curves, and those having a plurality of Foci; with remarks by Professor Forbes.* Communicated by Professor Forbes.

Mr Clerk Maxwell ingeniously suggests the extension of the common theory of the foci of the conic sections to curves of a higher degree of complication in the following manner:—

(1) As in the ellipse and hyperbola, any point in the curve has the *sum* or *difference* of two lines drawn from two points or *foci* = a constant quantity, so the author infers, that curves to a certain degree analogous, may be described and determined by the condition that the simple distance from one focus *plus* a multiple distance from the other, may be = a constant quantity; or more generally, *m* times the one distance + *n* times the other = constant.

(2) The author devised a simple mechanical means, by the wrapping of a thread round pins, for producing these curves. See Figs. 1 and 2. He

Fig. 1. Two Foci. Ratios 1, 2.

Fig. 2. Two Foci. Ratios 2, 3.

then thought of extending the principle to other curves, whose property should be, that the sum of the simple or multiple distances of any point of

VOL. I. 1

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1854 Smith's Prize Examination

8. If X, Y, Z be functions of the rectangular co-ordinates x, y, z, dS an element of any limited surface, l, m, n the cosines of the inclinations of the normal at dS to the axes, ds an element of the bounding line, shew that

$$\iint\left\{l\left(\frac{dZ}{dy}-\frac{dY}{dz}\right)+m\left(\frac{dX}{dz}-\frac{dZ}{dx}\right)+n\left(\frac{dY}{dx}-\frac{dX}{dy}\right)\right\}dS$$
$$=\int\left(X\frac{dx}{ds}+Y\frac{dy}{ds}+Z\frac{dz}{ds}\right)ds,$$

the differential coefficients of X, Y, Z being partial, and the single integral being taken all round the perimeter of the surface.

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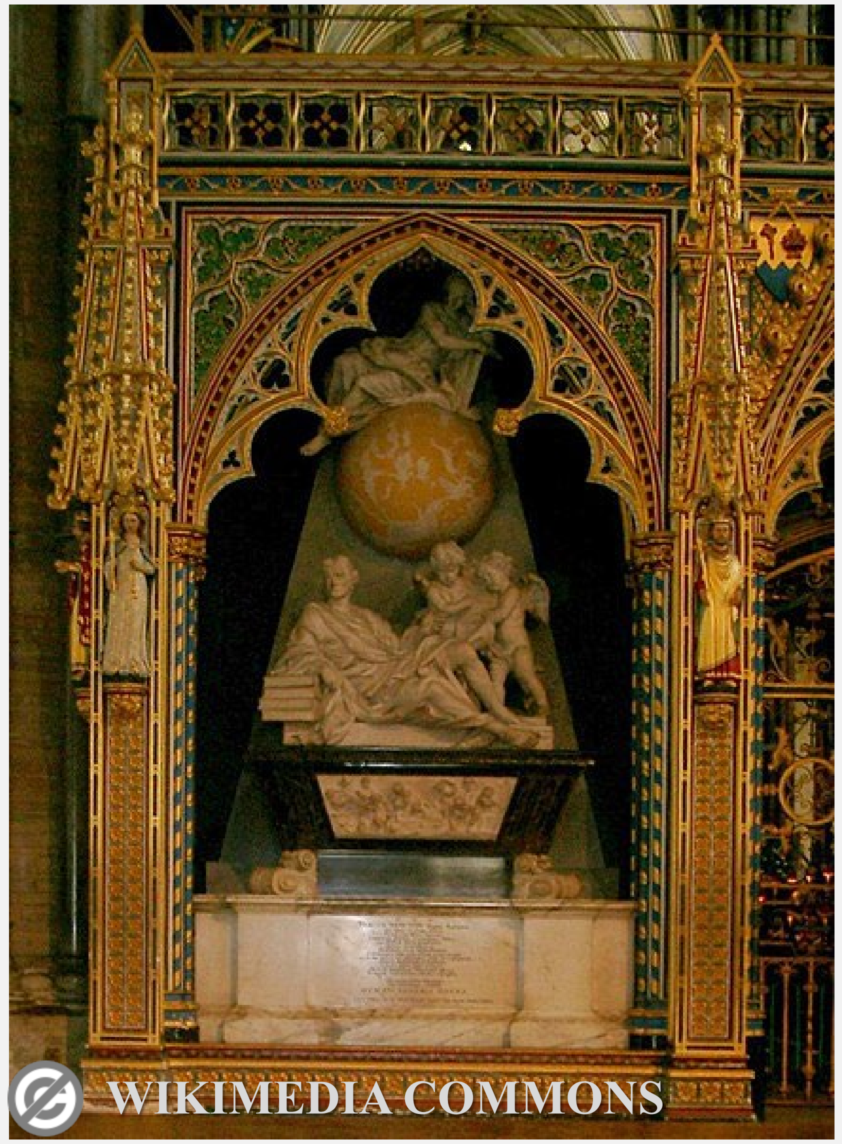
Memorials of Newton and Maxwell

Isaac Newton grave in Westminster Abbey



https://en.wikipedia.org/wiki/File:Isaac_Newton_grave_in_Westminster_Abbey.jpg

Maxwell's gravestone

https://commons.wikimedia.org/wiki/File:Maxwell%27s_gravestone.JPG

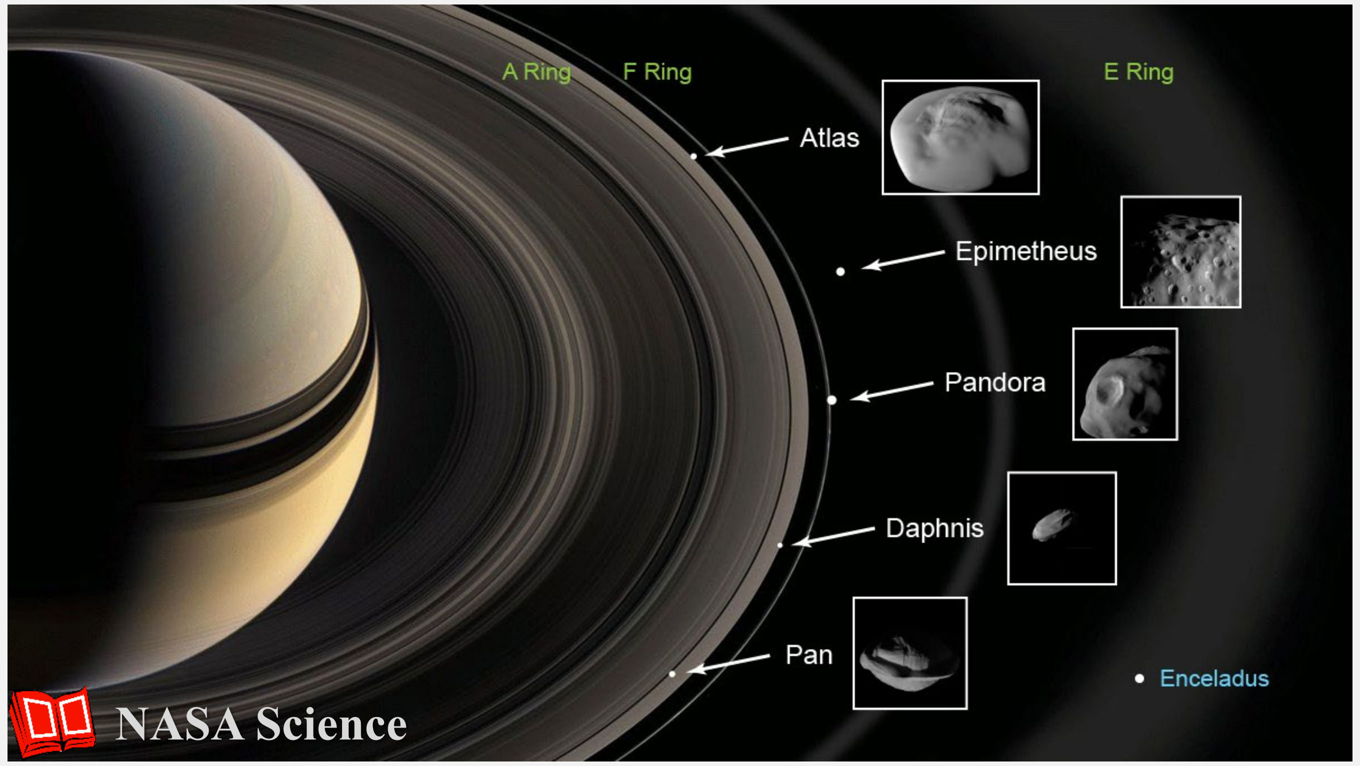
Memorial tablet of Maxwell in the nave of Westminster Abbey, London

James Clerk Maxwell memorial
This image can be purchased from Westminster Abbey Library
Image © 2022 Dean and Chapter of Westminster

https://www.westminster-abbey.org/abbey-commemorations/commemorations/james-clerk-maxwell

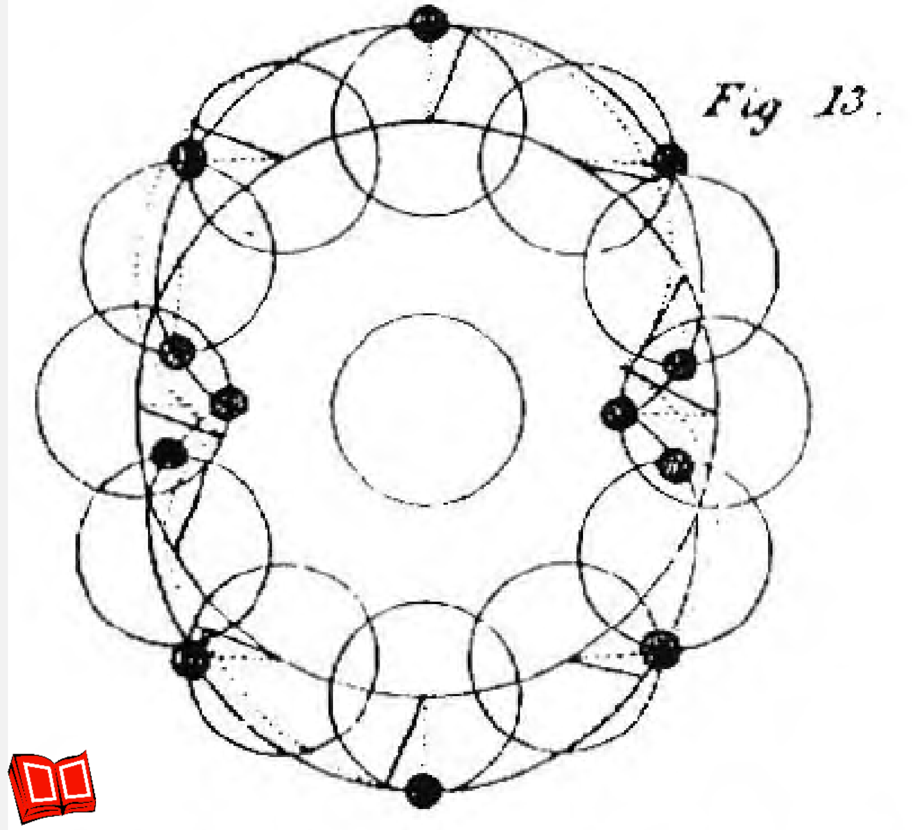
Major Research Topics Other Than Electromagnetics

Rings of Saturn

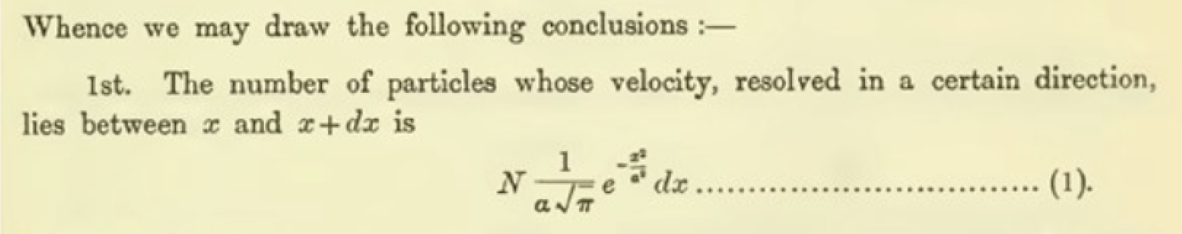


NASA Science

https://solarsystem.nasa.gov/news/886/cassini-finds-saturns-rings-coat-tiny-moons/



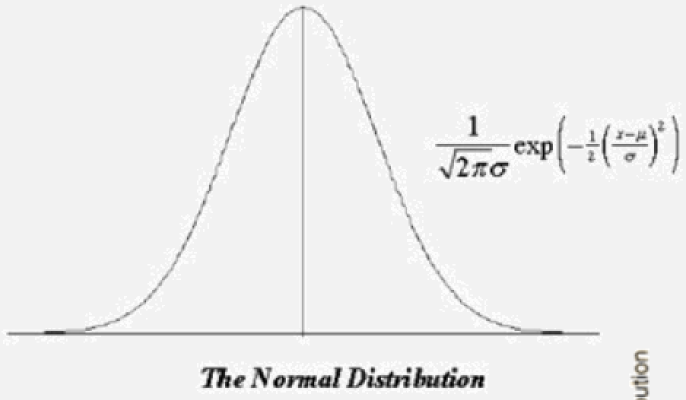
郭奕玲, 沈慧君編, 電磁場理論的奠基人—麥克斯韋, , 凡異出版社, 1999, 圖 1-17, p. 3.

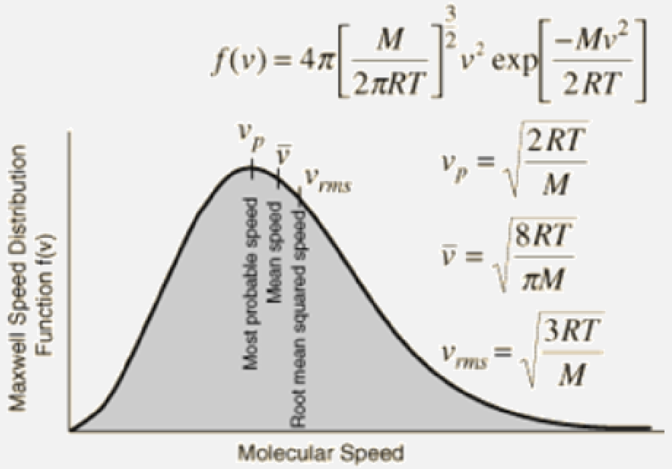


Whence we may draw the following conclusions :—

1st. The number of particles whose velocity, resolved in a certain direction, lies between x and $x+dx$ is

$$N\frac{1}{a\sqrt{\pi}}e^{-\frac{x^2}{a^2}}dx \quad \text{(1).}$$





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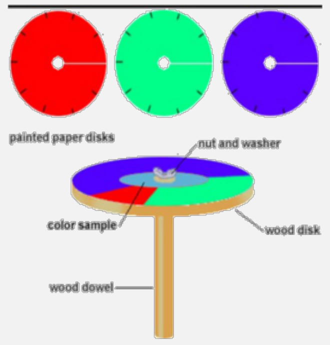
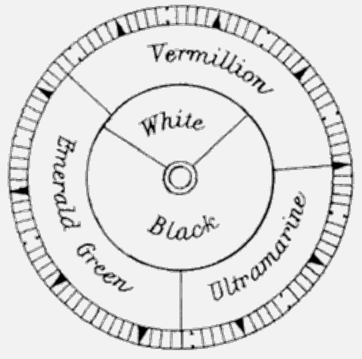
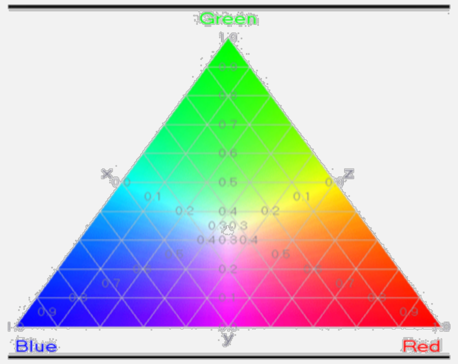
Colour Vision





SPIE. Digital Library



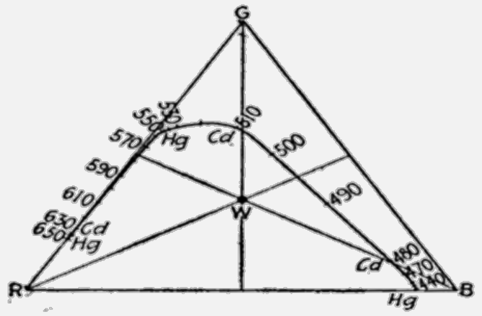


Fig. 31.—The Maxwell color-triangle.

SPIE. Digital Library

https://www.spiedigitallibrary.org/conference-proceedings-of-spie/11099/110990A/Maxwell-color-vision-and-the-color-triangle/10.1117/12.2529364.full?SSO=1

A Poem to Tomson’s Galvanometer

NATURE

[*May* 16, 1872

*A LECTURE ON THOMSON’S GALVANO-
METER*

DELIVERED TO A SINGLE PUPIL IN AN ALCOVE WITH
DRAWN CURTAINS

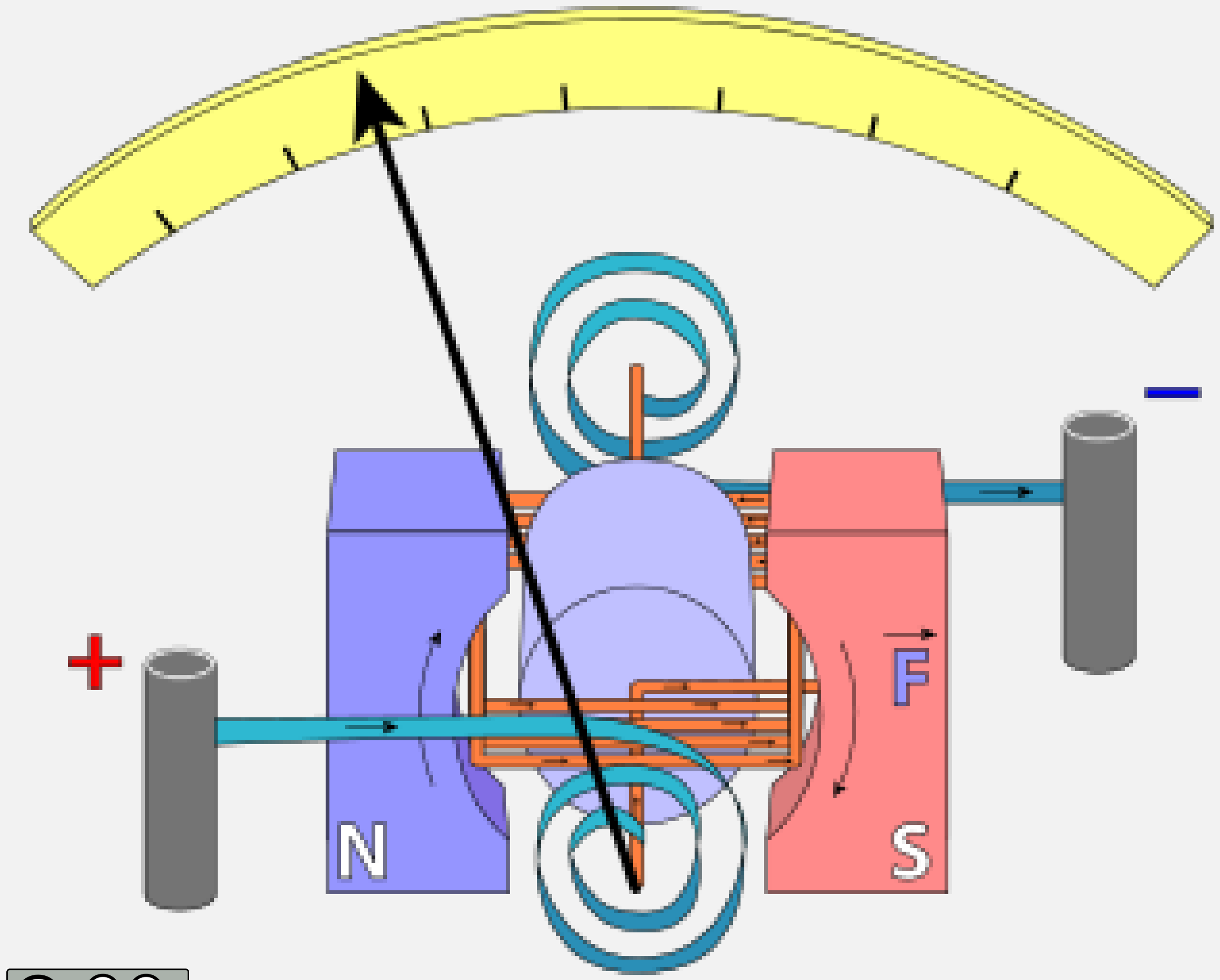
THE lamp-light falls on blackened walls,
And streams through narrow perforations;
The long beam trails o’er pasteboard scales,
With slow-decaying oscillations.
Flow, current! flow ! set the quick light-spot flying !
Flow, current ! answer, light-spot ! flashing, quivering,
dying.

O look ! how queer ! how thin and clear,
And thinner, clearer, sharper growing,
This gliding fire, with central wire
The fine degrees distinctly showing.
Swing, magnet! swing ! advancing and receding;
Swing, magnet! answer, dearest, what’s your final reading ?

O love! you fail to read the scale
Correct to tenths of a division;
To mirror heaven those eyes were given,
And not for methods of precision.
Break, contact ! break ! set the free light-spot flying !
Break, contact ! rest thee, magnet ! swinging, creeping,
dying.

dp/dt

Scottish Science Hall of Fame
https://digital.nls.uk/scientists/archive/74629736



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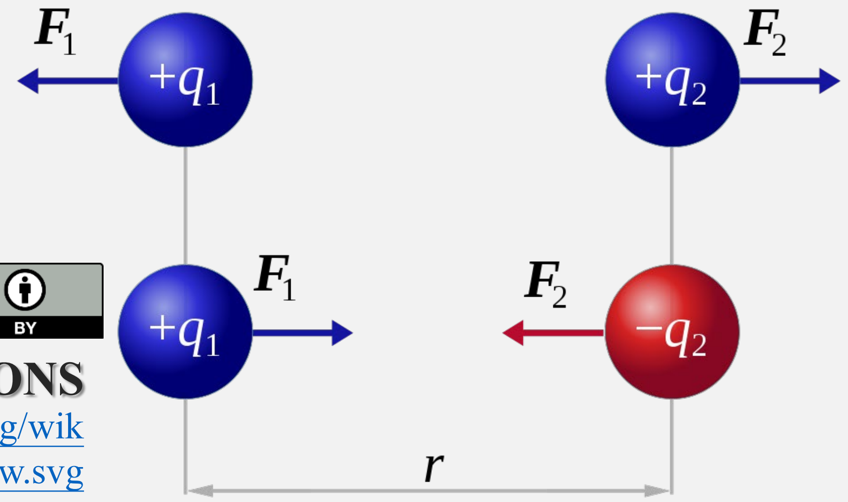
Action at a Distance: Coulomb's Law and Force between Currents

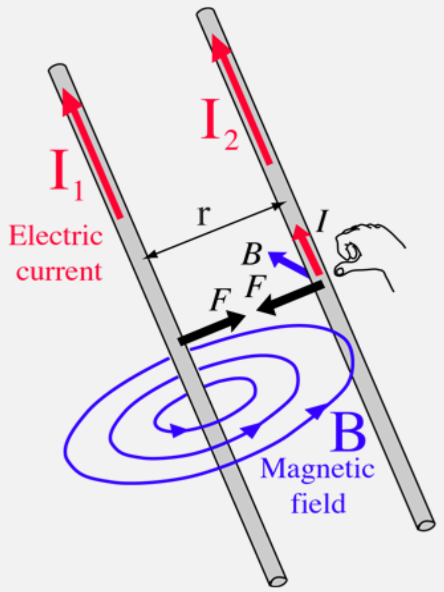
- Coulomb's law for electrostatics

$F = K \frac{q_1 q_2}{r^2}$, along the direction connecting positions of q_1 and q_2

- Force per unit length of parallel currents for magnetostatics

$F/L = K \frac{I_1 I_2}{r}$, along the direction perpendicular to the direction current and the direction connecting positions of I_1 *and* I_2 following the right-hand rule

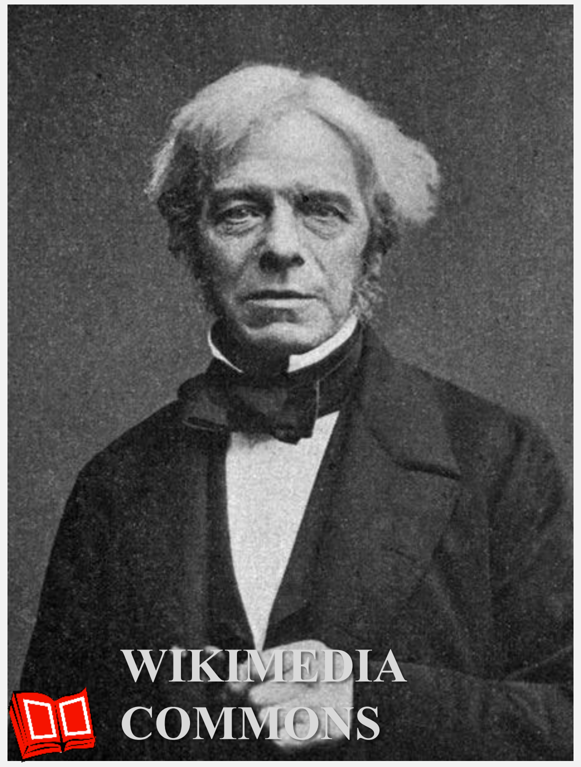




Faraday and His Law

- $\varepsilon = -\frac{\Delta\Phi_M}{\Delta t}$

Michael Faraday
1791-1867



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https://commons.wikimedia.org/wiki/File:Faraday-Millikan-Gale-1913.jpg

Faraday lecture
at Royal Institution 1856

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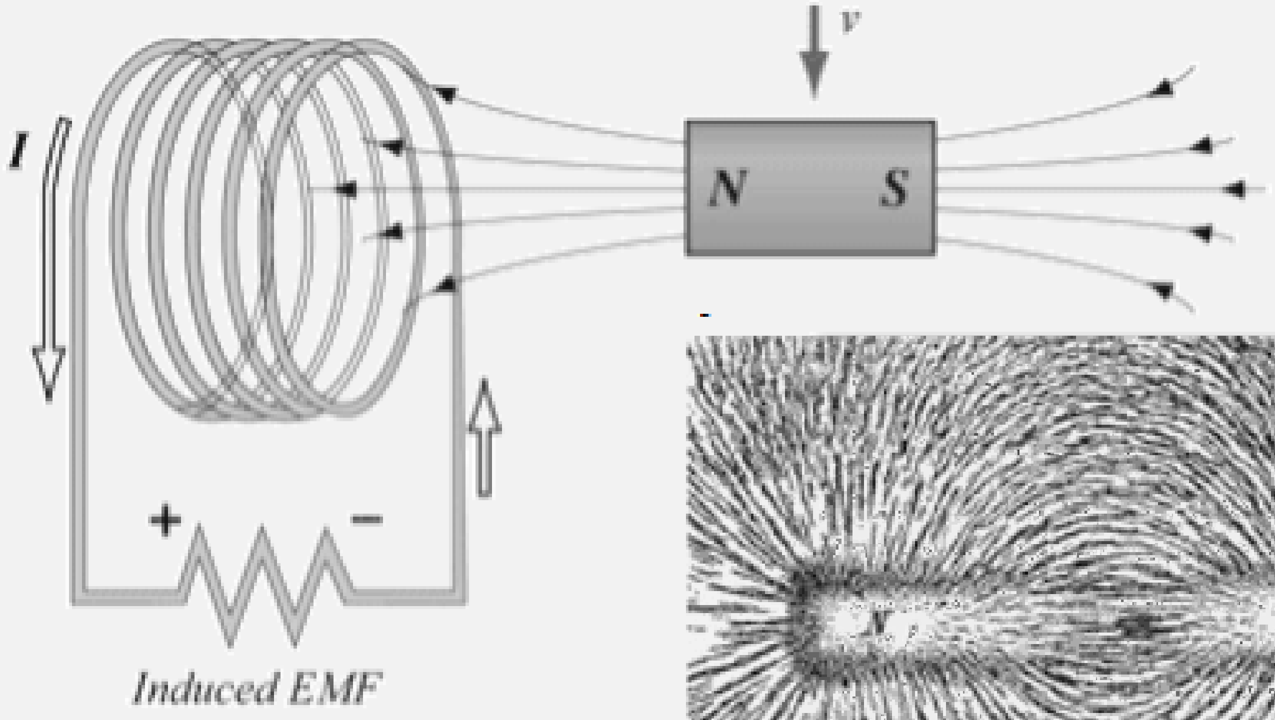
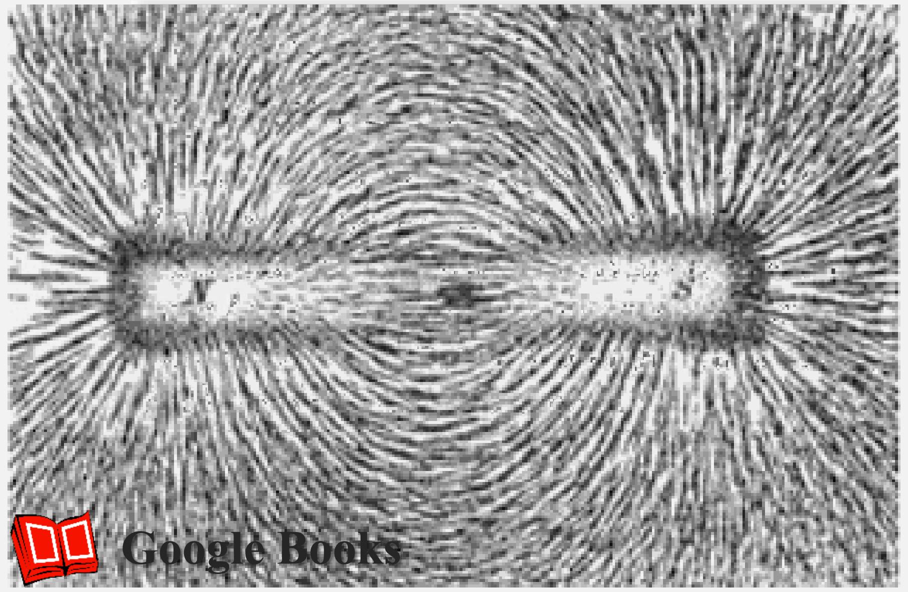
https://commons.wikimedia.org/wiki/File:Faraday_lecture_at_Royal_Institution_1856.jpg



Suffolk University

https://sites.suffolk.edu/kdshepard/2013/03/07/faradays-law/379a/



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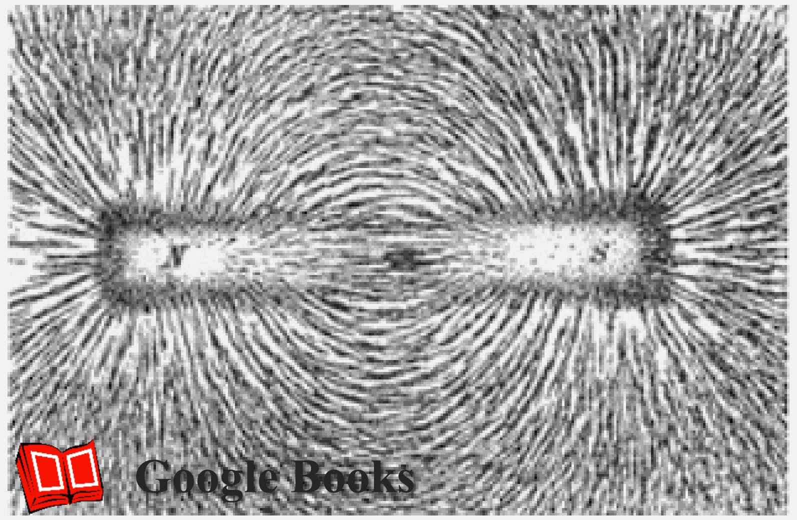
Faraday’s Lines of Force



https://books.google.com.tw/books?id=BT8AAAAAYAAJ&pg=PA242&redir_esc=y#v=onepage&q&f=false

- … by line of magnetic force, or magnetic line of force, … I mean that exercise of magnetic force which is exerted in the lines usually called magnetic curves, and which equally exist as passing from or to magnetic poles, or forming concentric circles round an electric current.
- By line of electric force, I mean the force exerted in the lines joining two bodies, acting on each other according to the principles of static electric induction (1161, &c), which may also be either in curved or straight lines.

Michael Faraday, Experimental Researches in Electricity, vol. III, London: Bernard Quaritch, 1855, ¶ 2149.

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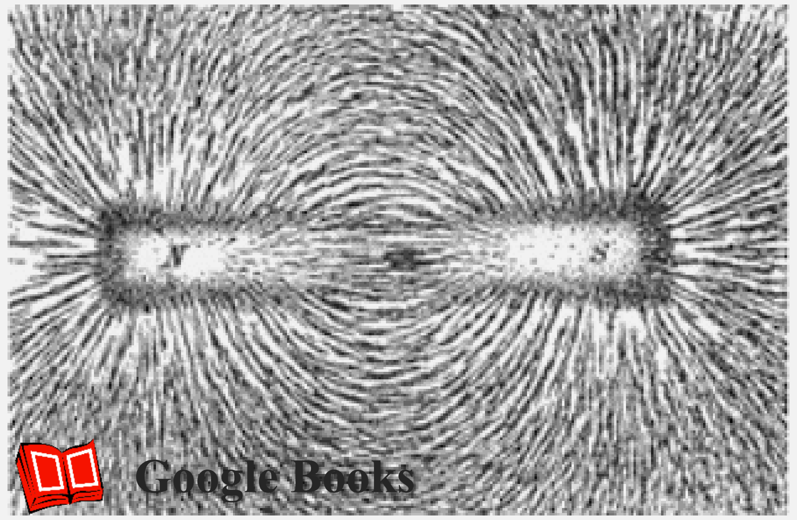
Physical Analogy

- By a physical analogy I mean that partial similarity between the laws of one science and those of another which makes each of them illustrate the other
- [A]ll the mathematics sciences are founded on relations between physical laws and laws of numbers, so that the exact science is to reduce the problems of nature to the determination of quantities by operations on numbers.

James Clerk Maxwell, On Faraday's Lines of Force, Transactions of the Cambridge Philosophical Society, vol. x, part 1, 1855. p. 155.

Analogy to Incompressible Fluid



https://books.google.com.tw/books?id=BT8AAAAAYAAJ&pg=PA242&redir_esc=y#v=onepage&q&f=false

- Line of force
  - "[T]his curve will indicate the direction of that force for every point through it which it passes, and may called on that account *a line of force* [Maxwell's emphasis]." (James Clerk Maxwell, On Faraday's Lines of Force, *Transactions of the Cambridge Philosophical Society*, vol. x, part 1, 1855. p. 158.)

James Clerk Maxwell, On Faraday's Lines of Force, Transactions of the Cambridge Philosophical Society, vol. x, part 1, 1855. p. 158.

- Tubes of incompressible fluid
  - "[C]onsider these curves not merely lines, but as fine tubes of variable section carrying an incompressible fluid, . . . We may make the velocity vary according to any given law, by regulating the section of the tube, and in this way we might represent the intensity of the force as well as its direction by the motion of the fluid in these tubes." (James Clerk Maxwell, On Faraday's Lines of Force, *Transactions of the Cambridge Philosophical Society*, vol. x, part 1, 1855. p. 159.)

James Clerk Maxwell, On Faraday's Lines of Force, Transactions of the Cambridge Philosophical Society, vol. x, part 1, 1855. p. 159.

Maxwell's Equation, 1865
Equation Names and Notations

Three equations of Magnetic Force	(B)
,, Electric Currents	(C)
,, Electromotive Force	(D)
,, Electric Elasticity	(E)
,, Electric Resistance	(F)
,, Total Currents	(A)
One equation of Free Electricity	(G)
,, Continuity	(H)

For Electromagnetic Momentum	F	G	H
,, Magnetic Intensity	α	β	γ
,, Electromotive Force	P	Q	R
,, Current due to true conduction	p	q	r
,, Electric Displacement	f	g	h
,, Total Current (including variation of displacement)	p'	q'	r'
,, Quantity of free Electricity	e		
,, Electric Potential	Ψ		

James Clerk Maxwell, "A dynamic theory of the electromagnetic field," Royal Society Transactions, vol. 155, 1865, pp. 459-512.

Maxwell's Equations (A),(B), (C): 1865

(A)

$$\left.\begin{aligned} p' &= p + \frac{df}{dt}, \\ q' &= q + \frac{dg}{dt}, \\ r' &= r + \frac{dh}{dt}, \end{aligned}\right\}$$

(B)

$$\left.\begin{aligned} \mu\alpha &= \frac{dH}{dy} - \frac{dG}{dz}, \\ \mu\beta &= \frac{dF}{dz} - \frac{dH}{dx}, \\ \mu\gamma &= \frac{dG}{dx} - \frac{dF}{dy}. \end{aligned}\right\}$$

(C)

$$\left.\begin{aligned} \frac{d\gamma}{dy} - \frac{d\beta}{dz} &= 4\pi p'. \\ \frac{d\alpha}{dz} - \frac{d\gamma}{dx} &= 4\pi q', \\ \frac{d\beta}{dx} - \frac{d\alpha}{dy} &= 4\pi r'. \end{aligned}\right\}$$

James Clerk Maxwell, "A dynamic theory of the electromagnetic field," Royal Society Transactions, vol. 155, 1865, pp. 459-512.

Maxwell's Equations (D),(E), (F), (G), (H): 1865

(D)

$$\left.\begin{aligned} P &= \mu\left(\gamma\frac{dy}{dt}-\beta\frac{dz}{dt}\right)-\frac{dF}{dt}-\frac{d\Psi}{dx}, \\ Q &= \mu\left(\alpha\frac{dz}{dt}-\gamma\frac{dx}{dt}\right)-\frac{dG}{dt}-\frac{d\Psi}{dy}, \\ R &= \mu\left(\beta\frac{dx}{dt}-\alpha\frac{dy}{dt}\right)-\frac{dH}{dt}-\frac{d\Psi}{dz}. \end{aligned}\right\}$$

(E)

$$\left.\begin{aligned} P &= kf, \\ Q &= kg, \\ R &= kh. \end{aligned}\right\}$$

(F)

$$\left.\begin{aligned} P &= -\varrho p, \\ Q &= -\varrho q, \\ R &= -\varrho r. \end{aligned}\right\}$$

(G)

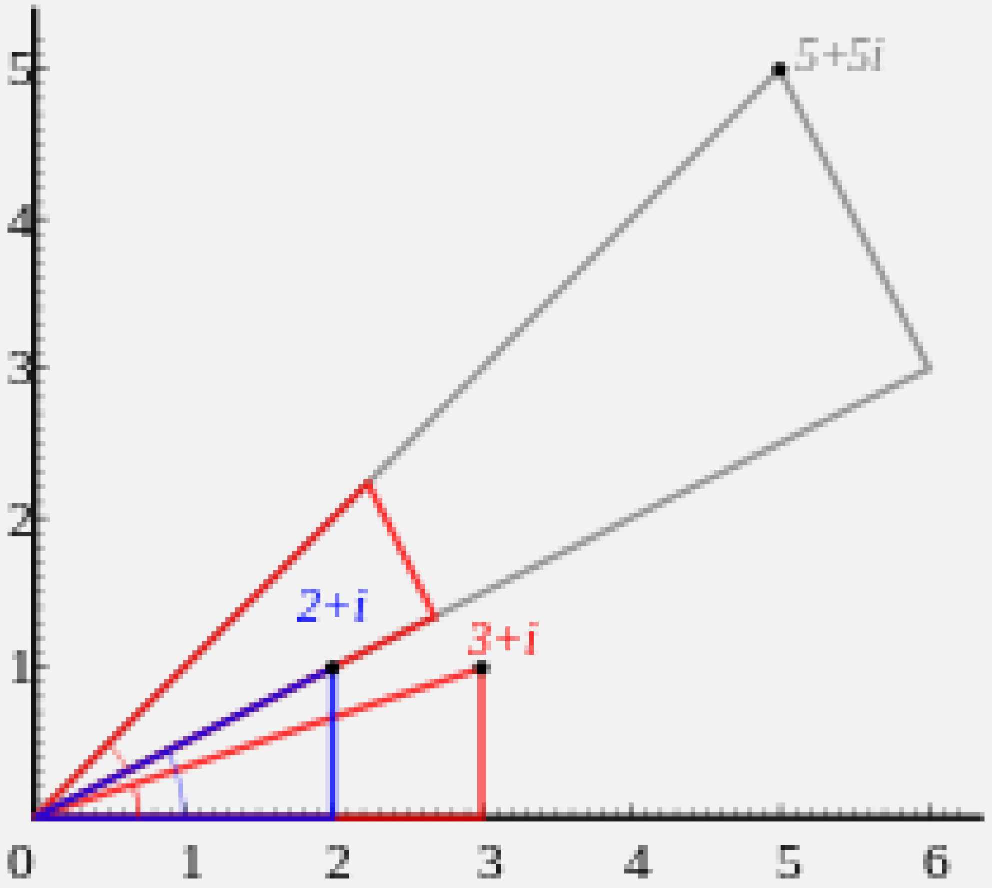
$$e+\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}=0.$$

(H)

$$\frac{de}{dt}+\frac{dp}{dx}+\frac{dq}{dy}+\frac{dr}{dz}=0$$

James Clerk Maxwell, "A dynamic theory of the electromagnetic field," Royal Society Transactions, vol. 155, 1865, pp. 459-512.

Complex Number and Quaternion



https://commons.wikimedia.org/wiki/File:Complex_multi.svg

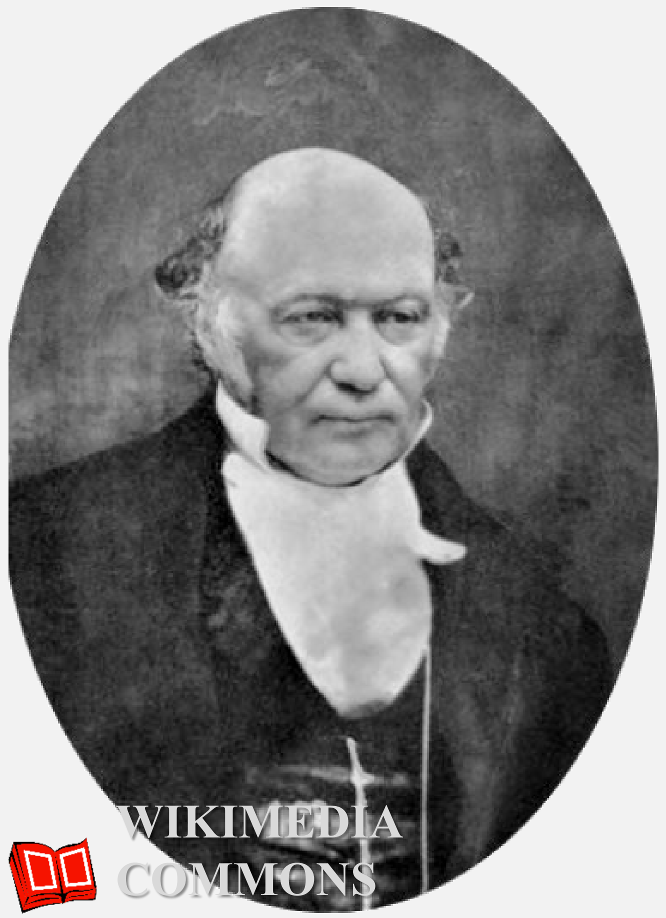
$$q = s + x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$$

Special case: $s = 0$

×	1	*i*	*j*	*k*
1	1	*i*	*j*	*k*
i	*i*	−1	*k*	−*j*
j	*j*	−*k*	−1	*i*
k	*k*	*j*	−*i*	−1

https://en.wikipedia.org/wiki/Quaternion

$$q_1\, q_2 = -(x_1x_2 + y_1y_2 + z_1z_2) + (y_1z_2 - z_1y_2)\mathbf{i} + (z_1x_2 - x_1z_2)\mathbf{j} + (x_1y_2 - x_2y_1)\mathbf{k}$$



Sir William Rowan Hamilton (1805–1865)

https://commons.wikimedia.org/wiki/File:William_Rowan_Hamilton_portrait_oval_combined.png

Maxwell's Equations, 1873 and Corresponding Modern Forms, (A),(B),(C)

$$a = \frac{dH}{dy} - \frac{dG}{dz}$$
$$b = \frac{dF}{dz} - \frac{dH}{dx}$$
$$c = \frac{dG}{dx} - \frac{dF}{dy}$$

(A) $\mathbf{B} = \nabla \times \mathbf{A}$

$$P = c\frac{dy}{dt} - b\frac{dz}{dt} - \frac{dF}{dt} - \frac{d\psi}{dx}$$
$$Q = a\frac{dz}{dt} - c\frac{dx}{dt} - \frac{dG}{dt} - \frac{d\psi}{dy}$$
$$R = b\frac{dx}{dt} - a\frac{dy}{dt} - \frac{dH}{dt} - \frac{d\psi}{dz}$$

(B) $\mathbf{E} = \mathbf{v} \times \mathbf{B} - \frac{\partial \mathbf{A}}{\partial t} - \nabla\phi$

$$X = vc - wb$$
$$Y = wa - uc$$
$$Z = ub - va$$

(C) $\mathbf{F} = \mathbf{J} \times \mathbf{B}$

Tapan K. Sarkar, etc., History of Wireless, New Jerse1y: John Wiley & Sons, Inc., 2006, Fig. 3.11, p. 185.
James Clerk Maxwell, A Treaties on Electricity and Magnetism, Oxford: Clarendon Press, 1873, pp. 480-493.

Maxwell's Equations, 1873 and Corresponding Modern Forms, (D),(E),(F),(G)

$$a = \alpha + 4\pi A$$
$$b = \beta + 4\pi B \qquad \text{(D)} \qquad \mathbf{B} = \mu_o \mathbf{H} + \mathbf{M}$$
$$c = \gamma + 4\pi C$$

$$4\pi u = \frac{d\gamma}{dy} - \frac{d\beta}{dz}$$
$$4\pi v = \frac{d\alpha}{dz} - \frac{d\gamma}{dx} \qquad \text{(E)} \qquad \mathbf{J} = \nabla \times \mathbf{H}$$
$$4\pi w = \frac{d\beta}{dx} - \frac{d\alpha}{dy}$$

$$\mathfrak{D} = \frac{1}{4\pi} K \mathfrak{E} \qquad \text{(F)} \qquad \mathbf{D} = \epsilon \mathbf{E}$$

$$\mathfrak{K} = C\mathfrak{E} \qquad \text{(G)} \qquad \mathbf{J}_c = \sigma \mathbf{E}$$

Tapan K. Sarkar, etc., History of Wireless, New Jerse1y: John Wiley & Sons, Inc., 2006, Fig. 3.11, p. 185.
James Clerk Maxwell, A Treaties on Electricity and Magnetism, Oxford: Clarendon Press, 1873, pp. 480-493.

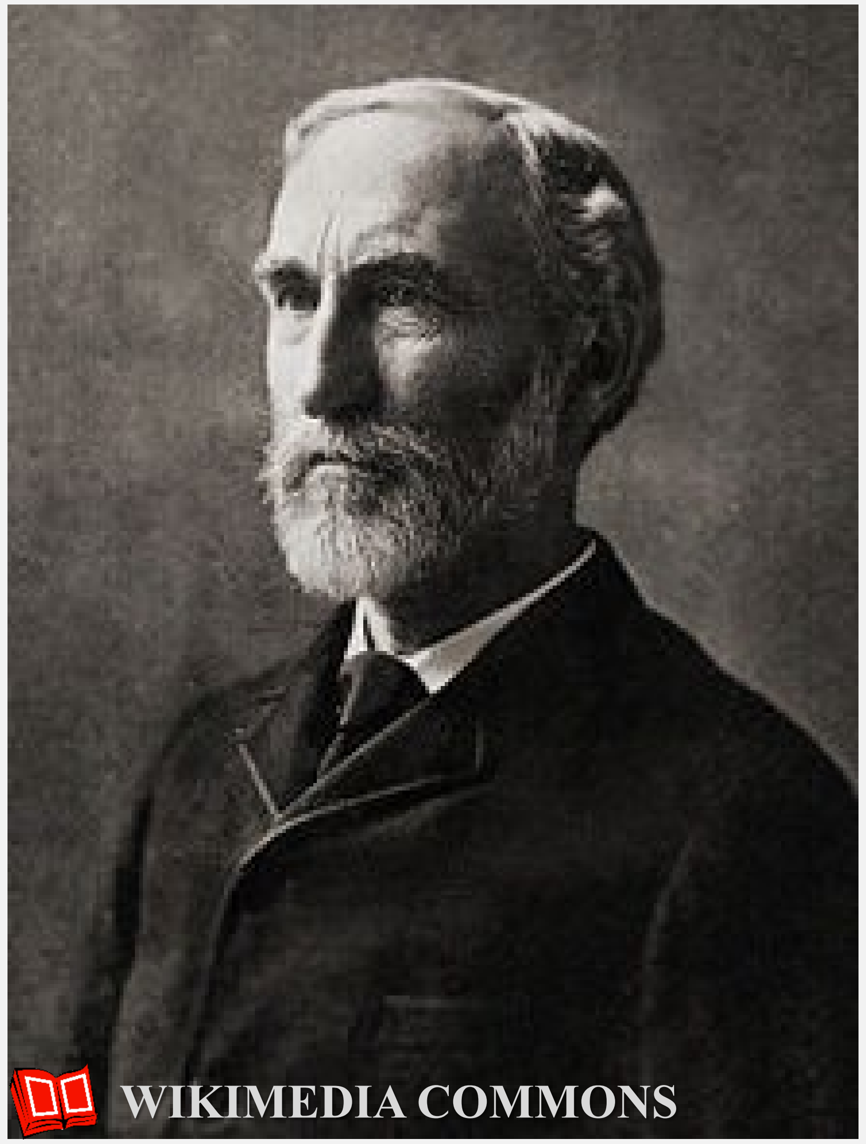
Maxwell’s Equations, 1873 and Corresponding Modern Forms,(H),(H*),(I),(I*), (J), (K), (L)

$\mathfrak{C} = (C + \frac{1}{4\pi} K \frac{d}{dt}) \mathfrak{E}$	(I)	$\mathbf{J} = \sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t}$
$u = CP + \frac{1}{4\pi} K \frac{dP}{dt}$		
$v = CQ + \frac{1}{4\pi} K \frac{dQ}{dt}$	(I*)	$\mathbf{J} = \sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t}$
$w = CR + \frac{1}{4\pi} K \frac{dR}{dt}$		
$\rho = \frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz}$	(J)	$\varrho = \nabla \cdot \mathbf{D}$
$\sigma = lf + mg + nh + l'f' + m'g' + n'h'$	(K)	$\varrho_s = \mathbf{n} \cdot (\mathbf{D}_1 - \mathbf{D}_2)$
$\mathfrak{B} = \mu \mathfrak{H}$	(L)	$\mathbf{B} = \mu \mathbf{H}$

Tapan K. Sarkar, etc., History of Wireless, New Jerse1y: John Wiley & Sons, Inc., 2006, Fig. 3.11, p. 185.
James Clerk Maxwell, A Treaties on Electricity and Magnetism, Oxford: Clarendon Press, 1873, pp. 480-493.

Vector Analysis: Oliver Heaviside and Josiah Willard Gibbs

Oliver Heaviside
1850–1925
https://commons.wikimedia.org/wiki/File:Oheaviside.jpg



Josiah Willard Gibbs
1839–1903
https://commons.wikimedia.org/wiki/File:Josiah_Willard_Gibbs_-from_MMS-.jpg

- Scalar product (Oliver Heaviside, *Electromagnetic Theory*, New York: Dover Publications, Inc., 1950, p. 38, ¶ 107.)
  - $\mathbf{AB} = AB \cos\theta$

Oliver Heaviside, Electromagnetic Theory, New York: Dover Publications, Inc., 1950, p. 38, ¶ 107.

- Vector product (Oliver Heaviside, *Electromagnetic Theory*, New York: Dover Publications, Inc., 1950, p. 40, ¶ 111.)
  - $\mathbf{C} = V\mathbf{AB}, C = AB \sin\theta$

Oliver Heaviside, Electromagnetic Theory, New York: Dover Publications, Inc., 1950, p. 40, ¶ 111.

Oliver Heaviside’s Circuitation, curl, and divergence, 1885, 1893

- Circuitation (Oliver Heaviside, *Electromagnetic Theory*, New York: Dover Publications, Inc., 1950, p. 9, ¶ 33.)
  - “used,…, to indicate the often-occurring operation of a line-integral in a closed circuit”

Oliver Heaviside, Electromagnetic Theory, New York: Dover Publications, Inc., 1950, p. 9, ¶ 33.

- curl (Oliver Heaviside, *Electromagnetic Theory*, New York: Dover Publications, Inc., 1950, p. 9, ¶ 36.)
  - “The circuitation of the force is then called its ‘curl’.”

Oliver Heaviside, Electromagnetic Theory, New York: Dover Publications, Inc., 1950, p. 9, ¶ 36.

- divergence (Oliver Heaviside, *Electromagnetic Theory*, New York: Dover Publications, Inc., 1950, p. 13, ¶ 51.)
  - “The divergence of any flux is the amount of the flux leaving the unit volume.”

Oliver Heaviside, Electromagnetic Theory, New York: Dover Publications, Inc., 1950, p. 13, ¶ 51.

Oliver Heaviside's Duplex Form of Maxwell's Equations, 1885, 1893

- $\mathbf{D} = \epsilon\mathbf{E}$
- $\mathbf{B} = \mu\mathbf{H}$
- $\text{curl}\,\mathbf{H} = \mathbf{J}$
- $-\text{curl}\,\mathbf{E} = \mathbf{G}$
- $\text{div}\,\mathbf{D} = \rho$
- $\text{div}\,\mathbf{B} = \sigma$

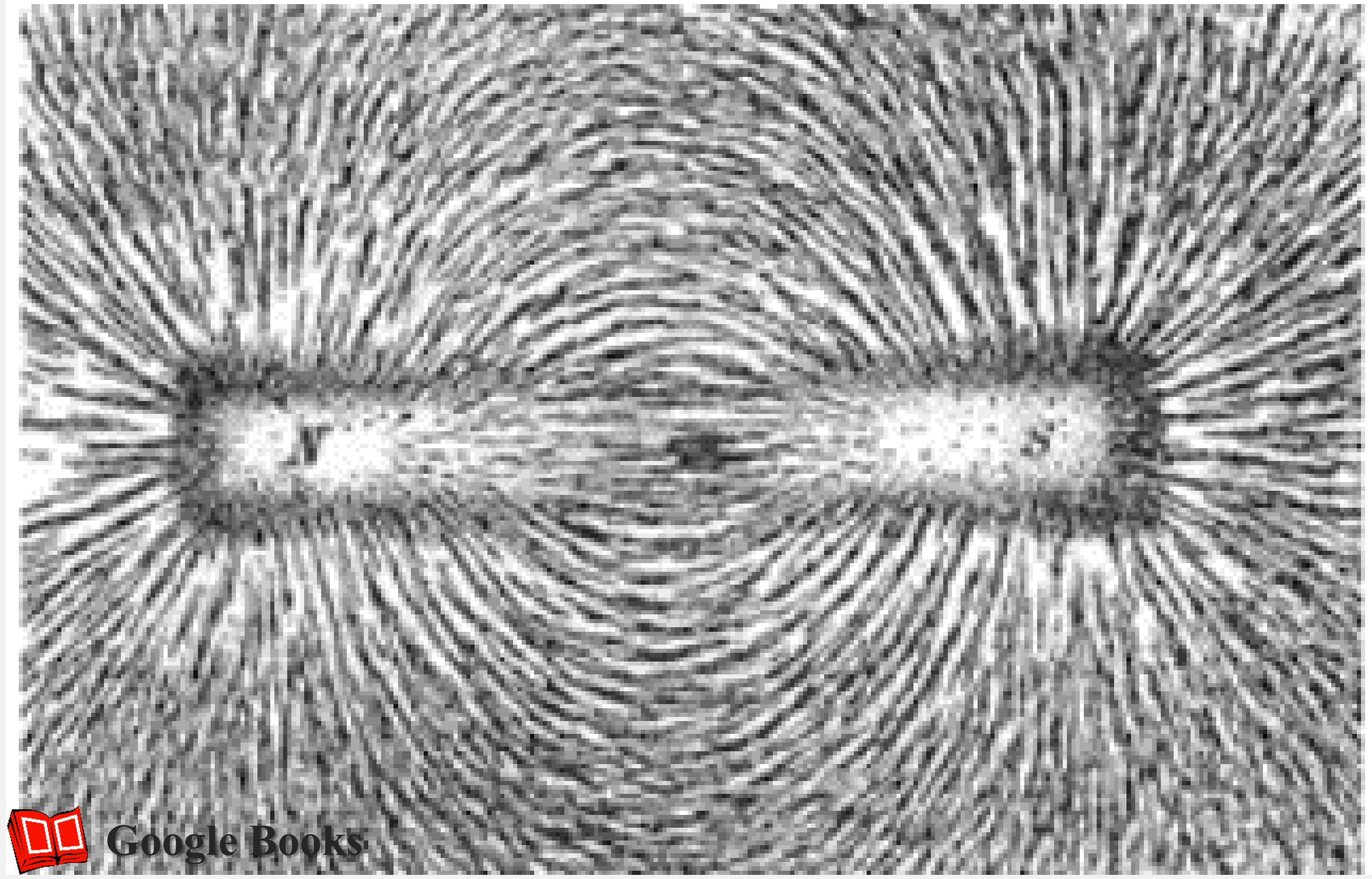
Maxwell's Equations, Modern Text Version

- $\nabla \cdot \bar{D}(\bar{r}, t) = \rho_e(\bar{r}, t)$
- $\nabla \cdot \bar{B}(\bar{r}, t) = 0$
- $\nabla \times \bar{E}(\bar{r}, t) = -\frac{\partial}{\partial t}\bar{B}(\bar{r}, t)$
- $\nabla \times \bar{H}(\bar{r}, t) = \bar{J}(\bar{r}, t) + \frac{\partial}{\partial t}\bar{D}(\bar{r}, t)$

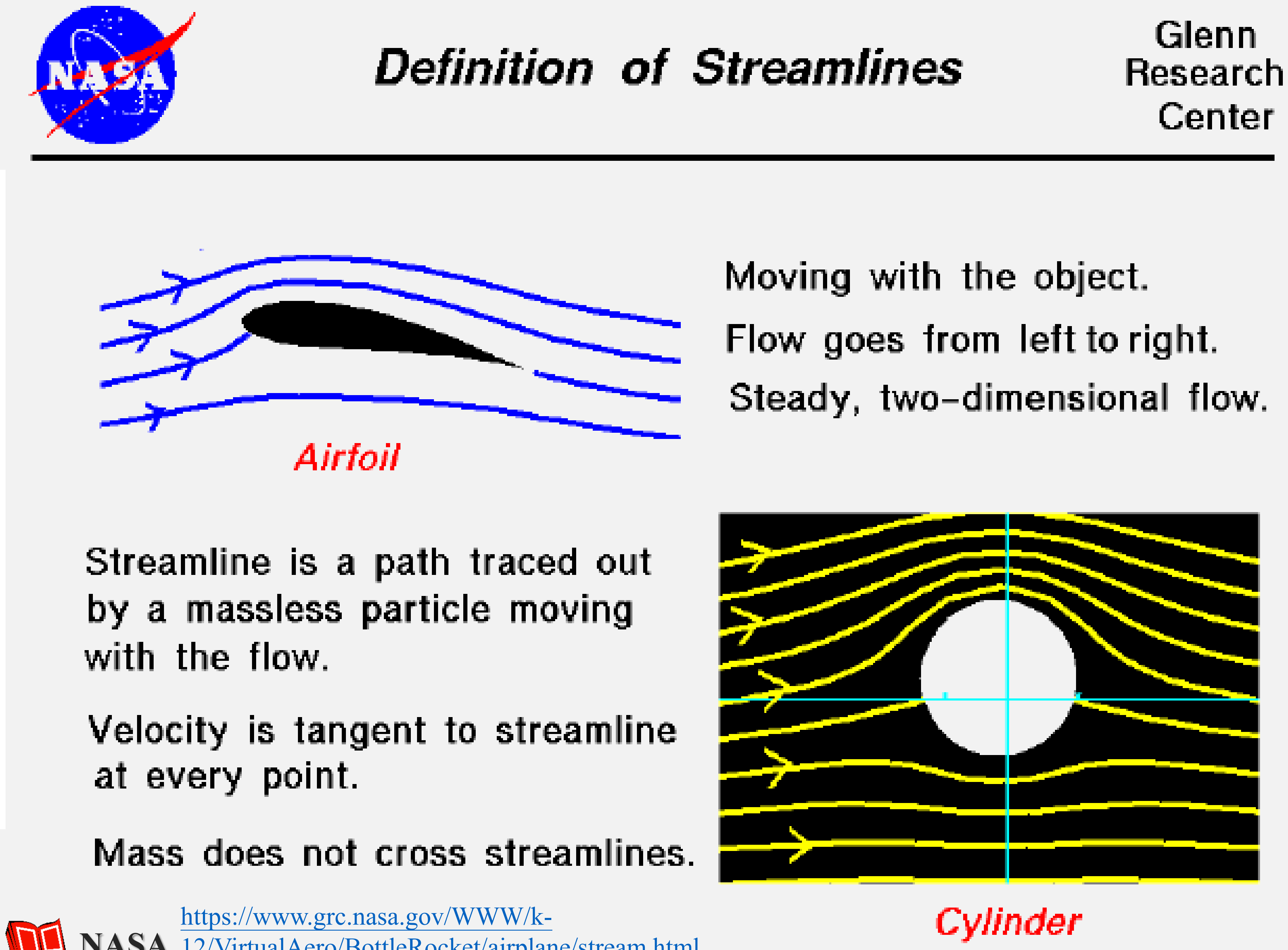
Outline

- Life of James Clark Maxwell
- Electricity and Magnetism before Maxwell
- Evolution of Maxwell's Equations
- "Derivation" of Maxwell's Equations

Lines of Force and Stream Lines

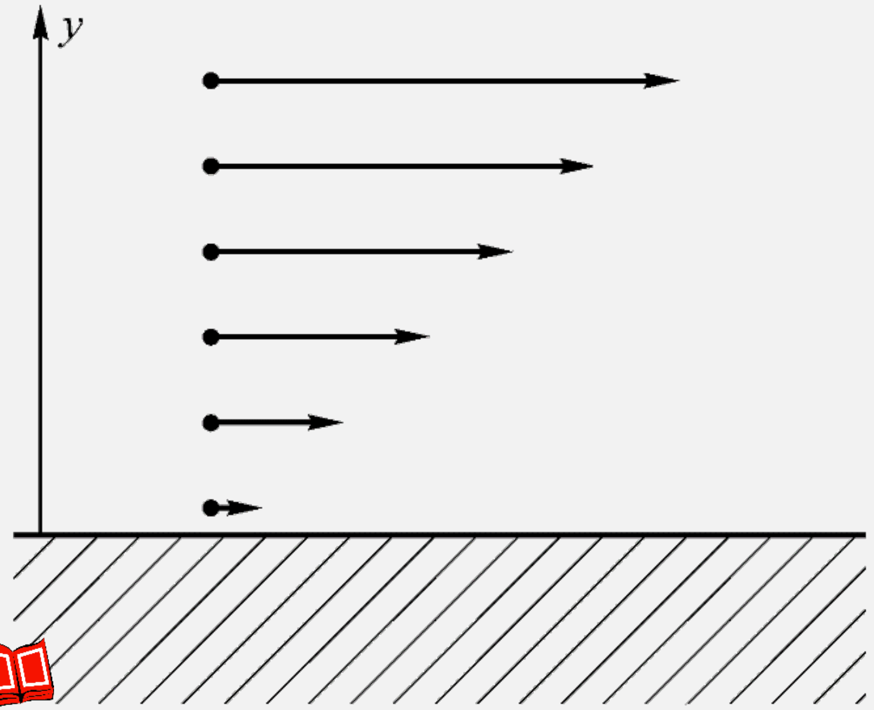


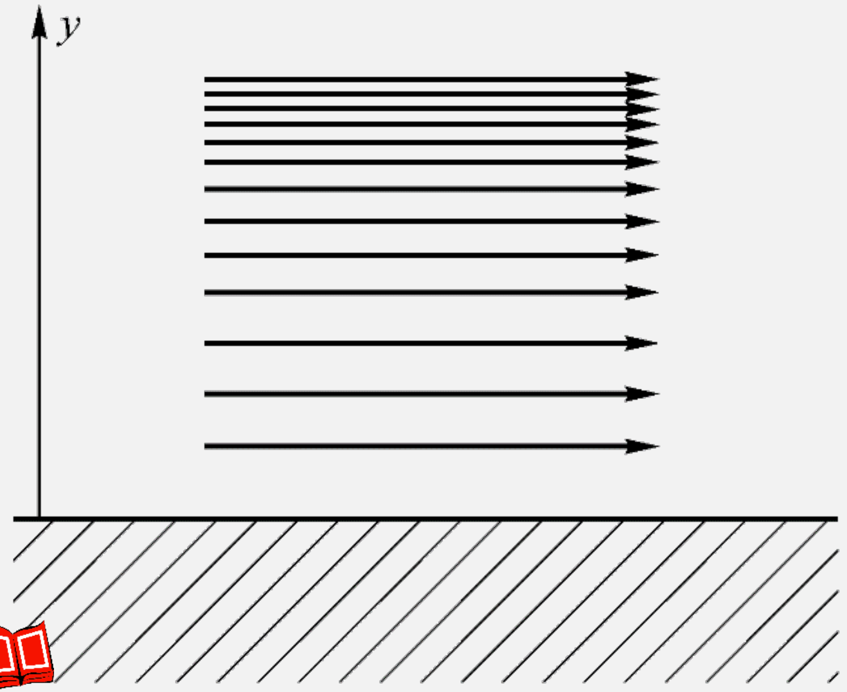
Google Books
https://books.google.com.tw/books?id=BT8AAAAAYAAJ&pg=PA242&redir_esc=y#v=onepage&q&f=false



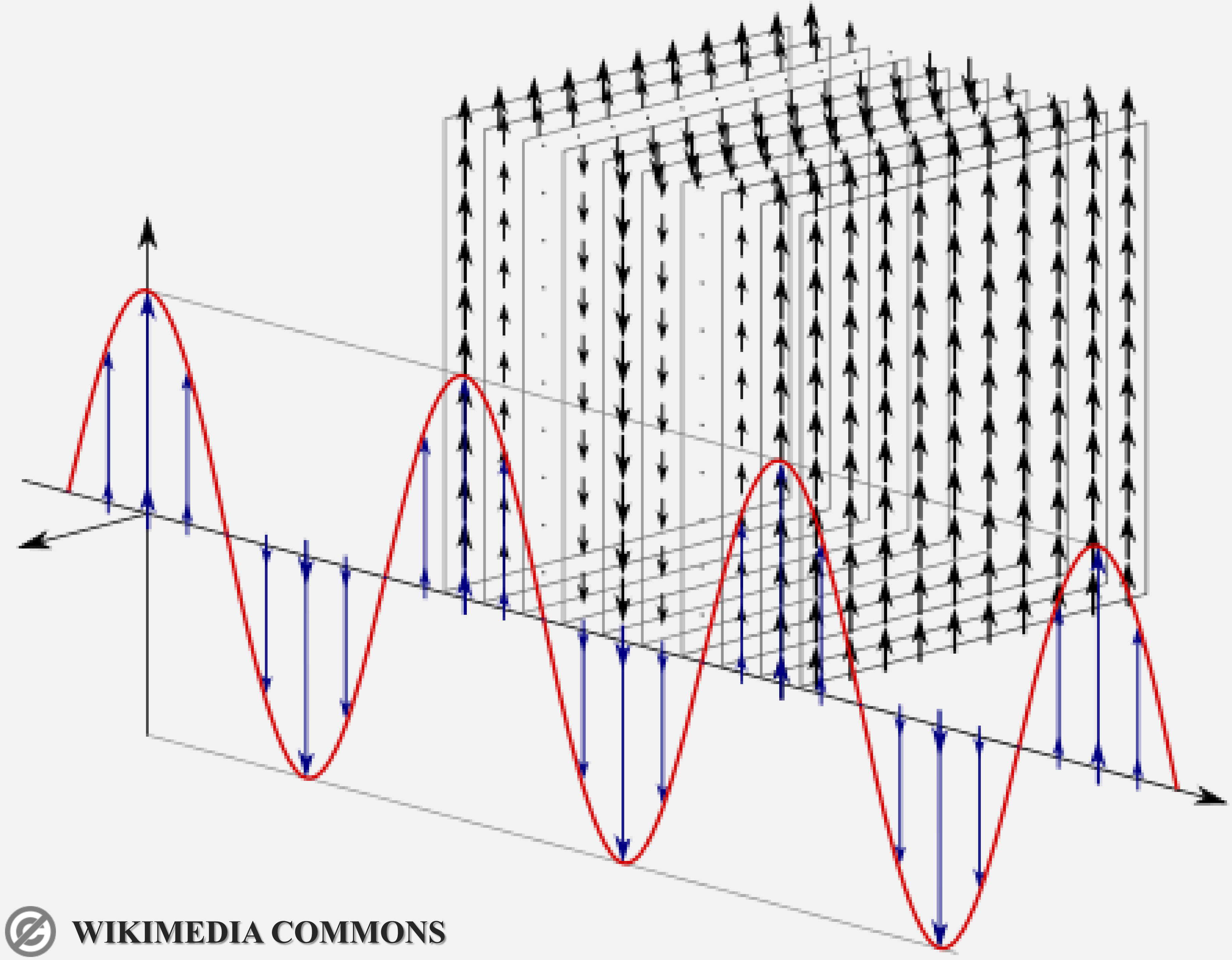
NASA https://www.grc.nasa.gov/WWW/k-12/VirtualAero/BottleRocket/airplane/stream.html

Expressions of Field Strength with Lines of Field





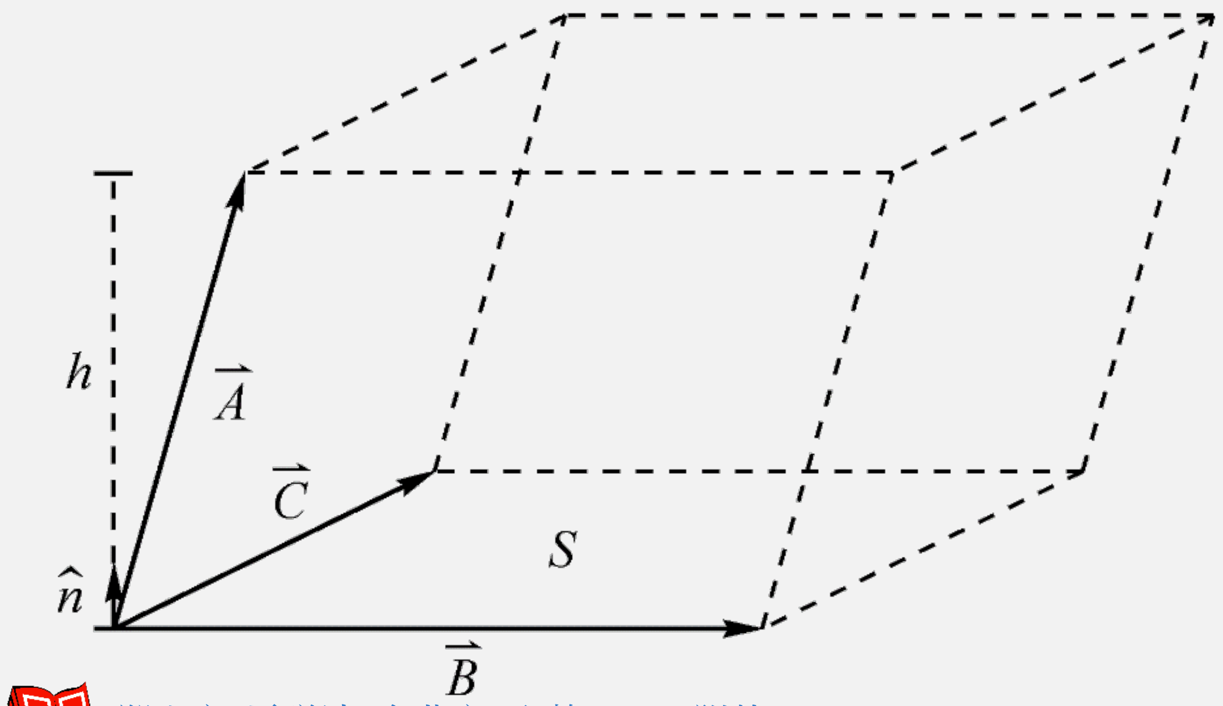
鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.



WIKIMEDIA COMMONS
https://commons.wikimedia.org/wiki/File:Linear_Polarization_Linearly_Polarized_Light_plane_wave.svg

Vector, Products, and Magnitude

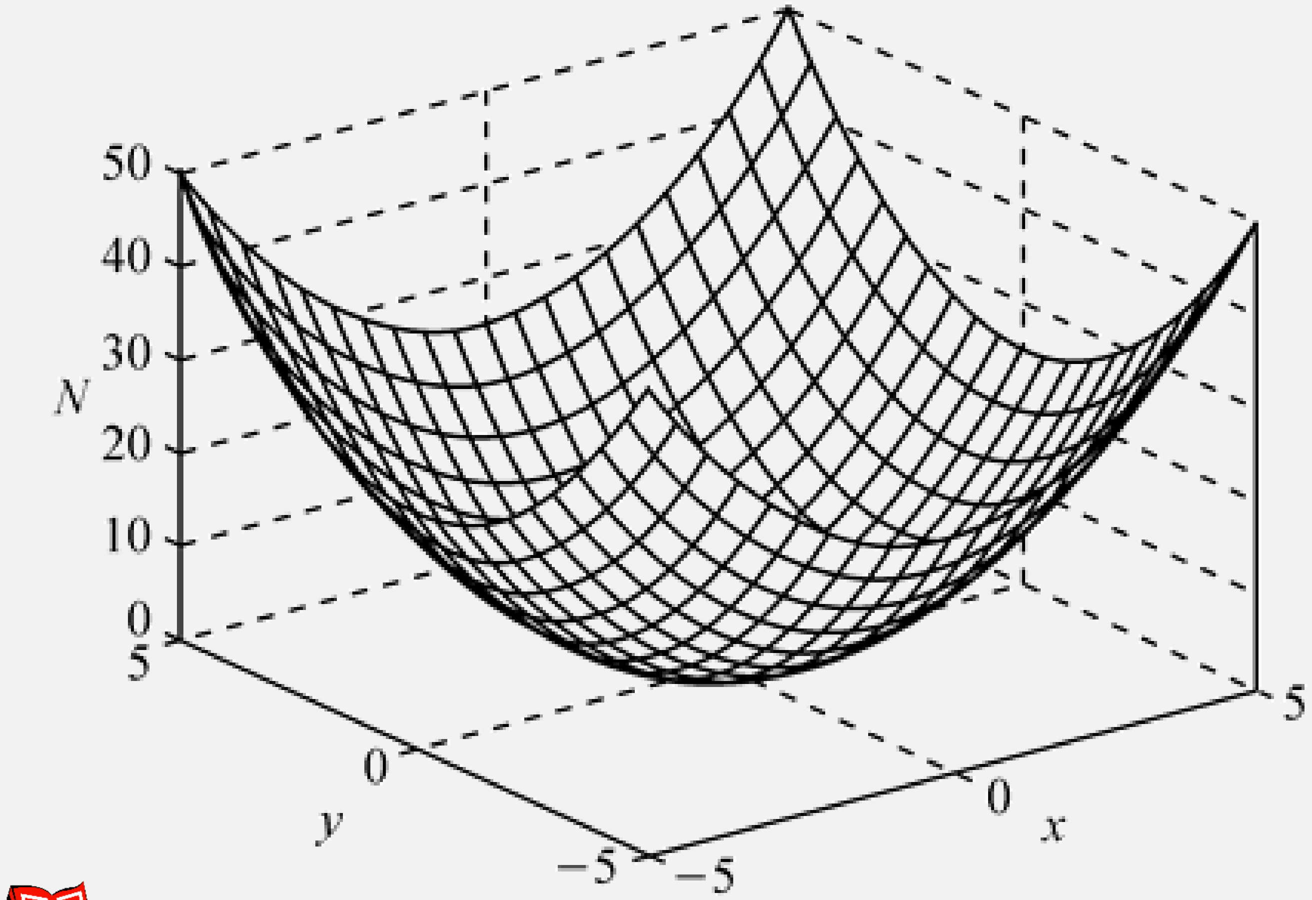
- $\bar{V} = [V_x \quad V_y \quad V_z]^T = \hat{x}V_x + \hat{y}V_y + \hat{z}V_z$
- $|\bar{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2}$
- $\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z = |\bar{A}||\bar{B}| \cos\theta_{AB}$
- $|\bar{V}| = \sqrt{\bar{V} \cdot \bar{V}}$
- $\bar{B} \times \bar{C} = \hat{x}(B_y C_z - B_z C_y) + \hat{y}(B_z C_x - B_x C_z) + \hat{z}(B_x C_y - B_y C_x)$
- $\bar{B} \times \bar{C} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \hat{n}|\bar{B}||\bar{C}| \sin\theta_{BC}$



鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.

Position Vector and Surface

- $\bar{r} = \hat{x}x + \hat{y}y + \hat{z}z$
- $\bar{r} = \bar{r}(u,v) = \hat{x}x(u,v) + \hat{y}y(u,v) + \hat{z}z(u,v)$
- Example: $\bar{r}(u,v) = \hat{x}u + \hat{y}v + \hat{z}\,(u^2 + v^2)$
- $\hat{n}(x,y,z) = \hat{n}(u,v) =$
- $\hat{x}n_x + \hat{y}n_y + \hat{z}n_z =$
- $\frac{\partial\bar{r}(u,v)}{\partial u} \times \frac{\partial\bar{r}(u,v)}{\partial v} / Jacobian$
- $Jacobian = \left|\frac{\partial\bar{r}(u,v)}{\partial u} \times \frac{\partial\bar{r}(u,v)}{\partial v}\right|$



鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.

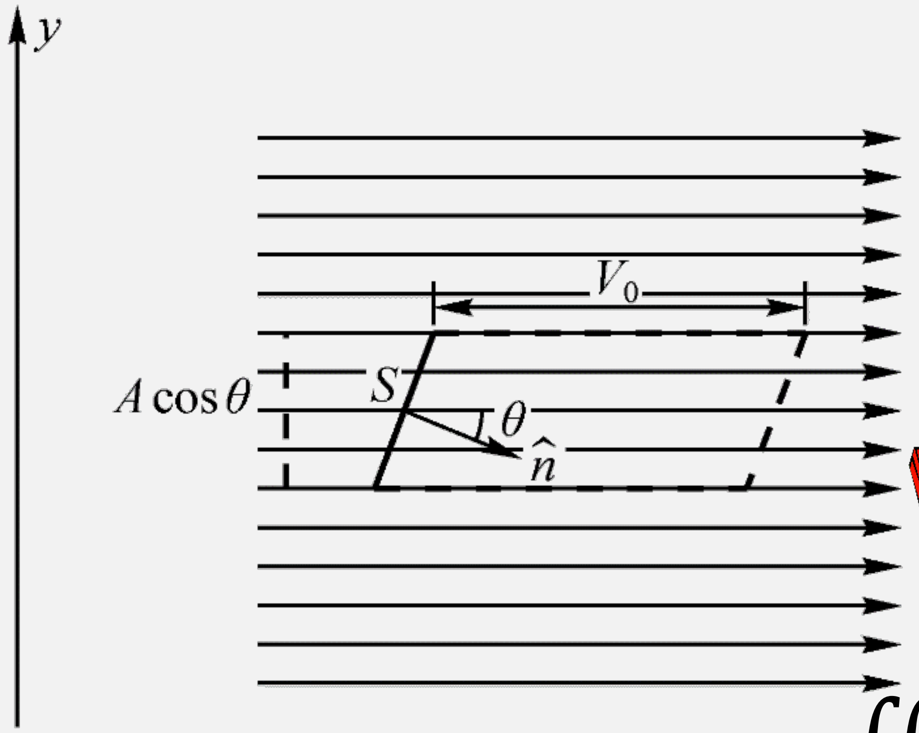
Flux and Flux Density

- Flux density

$$\bar{V}(x,y,z,t) = \hat{x}V_x(x,y,z,t) + \hat{y}V_y(x,y,z,t) + \hat{z}V_z(x,y,z,t)$$

- Flux through surface S

$$\Phi(t) = \int_S \bar{V}(x,y,z,t) \cdot \hat{n}da$$



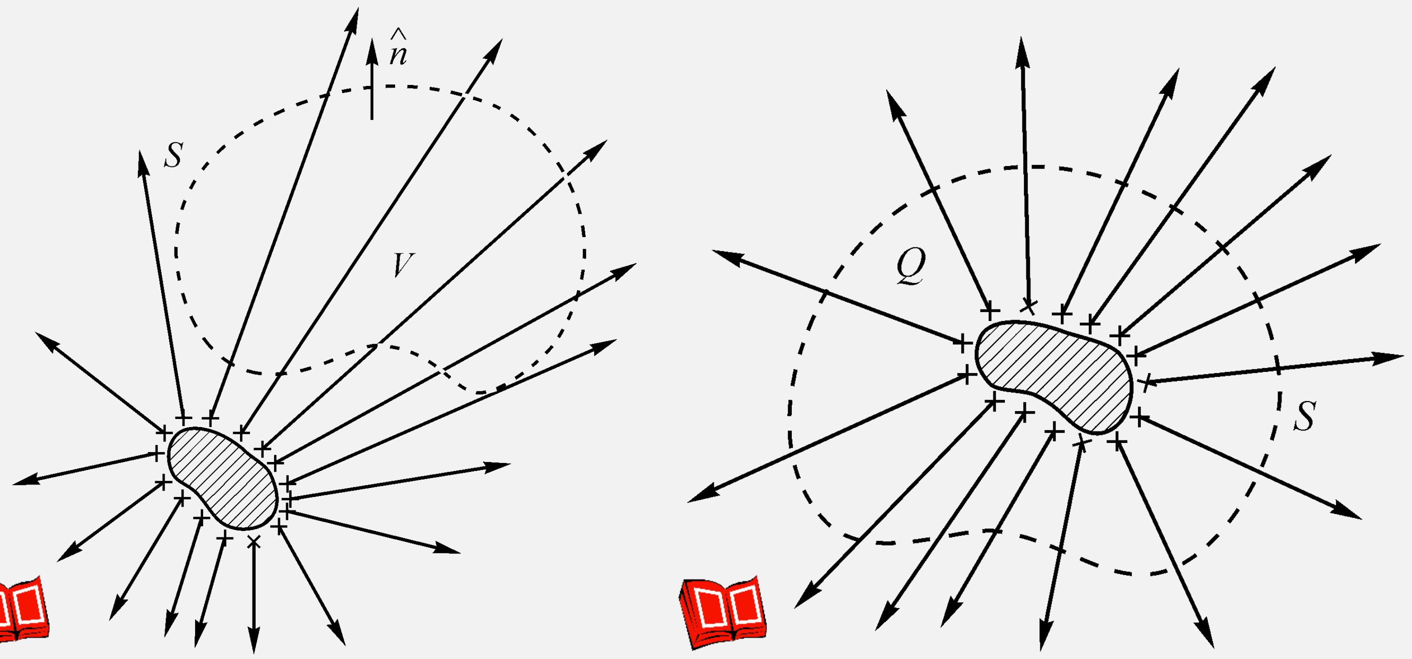
鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.

- $\Phi(t) =$

$$\iint_{S_{yz}} V_x(x,y,z,t)n_x dydz + \iint_{S_{zx}} V_y(x,y,z,t)n_y dzdx + \iint_{S_{xy}} V_z(x,y,z,t)n_z dxdy$$

Electric Flux Density and Gauss's Law

- Lines of electric field
- Imaginary substance in the analogy to an imaginary incompressible fluid
- Electric flux Φ_E and electric charge
- Flux tube
- Electric Flux Density $\overline{D}(x, y, z, t)$
- Gauss's law



鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.

$$Q(t) = \oint_S \overline{D}(x, y, z, t) \cdot \hat{n} da =$$

$$\iint_{S_{yz}} D_x(x, y, z, t) n_x dydz + \iint_{S_{zx}} D_y(x, y, z, t) n_y dzdx + \iint_{S_{xy}} D_z(x, y, z, t) n_z dxdy$$

Divergence (1/2)

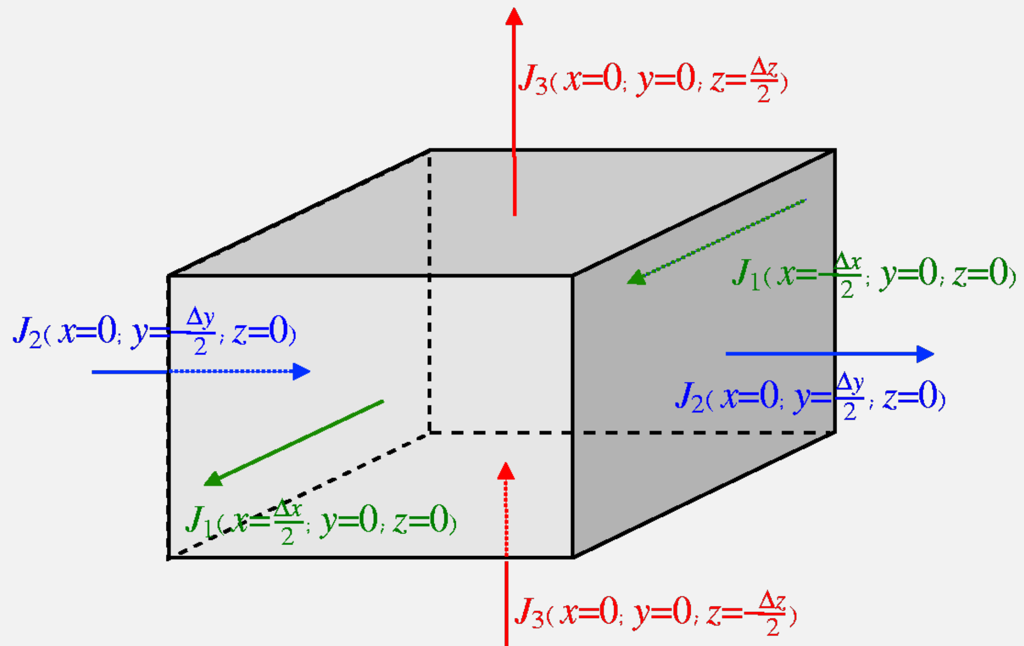
- $\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f\left(x+\frac{\Delta x}{2}\right)-f\left(x-\frac{\Delta x}{2}\right)}{\Delta x} \approx \frac{f\left(x+\frac{\Delta x}{2}\right)-f\left(x-\frac{\Delta x}{2}\right)}{\Delta x}$
- Flux density

$$\bar{V}(x,y,z,t) = [J_1(x,y,z,t) \quad J_2(x,y,z,t) \quad J_3(x,y,z,t)]^T$$

- Net increase of material in the x-direction

$$\left\{J_1\left(x+\frac{\Delta x}{2},y,z,t\right) - J_1\left(x-\frac{\Delta x}{2},y,z,t\right)\right\}\Delta y \Delta z \approx$$

$$\frac{J_1\left(x+\frac{\Delta x}{2},y,z,t\right)-J_1\left(x-\frac{\Delta x}{2},y,z,t\right)}{\Delta x}\Delta x \Delta y \Delta z$$



Massachusetts Institute of Technology
http://pruffle.mit.edu/3.016-2005/Lecture_16_web/node2.html

Divergence (2/2)

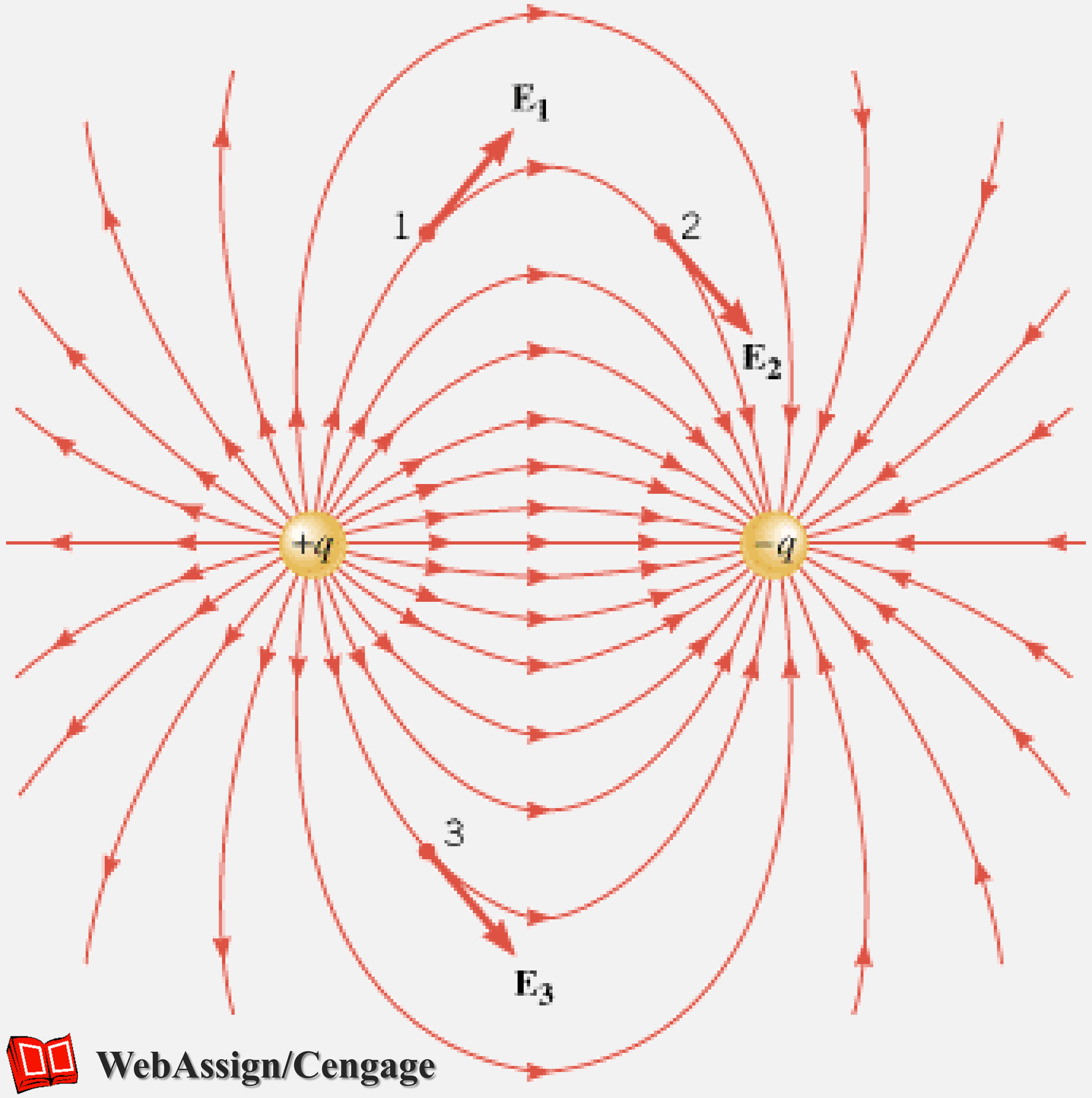
- Net increase of material per unit volume from the small cube
- $div\,\bar{V}(x,y,z,t) = \lim_{\Delta\tau\to 0}\left\{\frac{\Delta J_1(x,y,z,t)}{\Delta x} + \frac{\Delta J_2(x,y,z,t)}{\Delta y} + \frac{\Delta J_3(x,y,z,t)}{\Delta z}\right\} =$
- $\frac{\partial J_1(x,y,z,t)}{\partial x} + \frac{\partial J_2(x,y,z,t)}{\partial y} + \frac{\partial J_3(x,y,z,t)}{\partial z}$
- With $\bar{V}(x,y,z,t) = \hat{x}V_x(x,y,z,t) + \hat{y}V_y(x,y,z,t) + \hat{z}V_z(x,y,z,t)$,

  $$div\,\bar{V}(x,y,z,t) = \frac{\partial V_x(x,y,z,t)}{\partial x} + \frac{\partial V_y(x,y,z,t)}{\partial y} + \frac{\partial V_z(x,y,z,t)}{\partial z} =$$

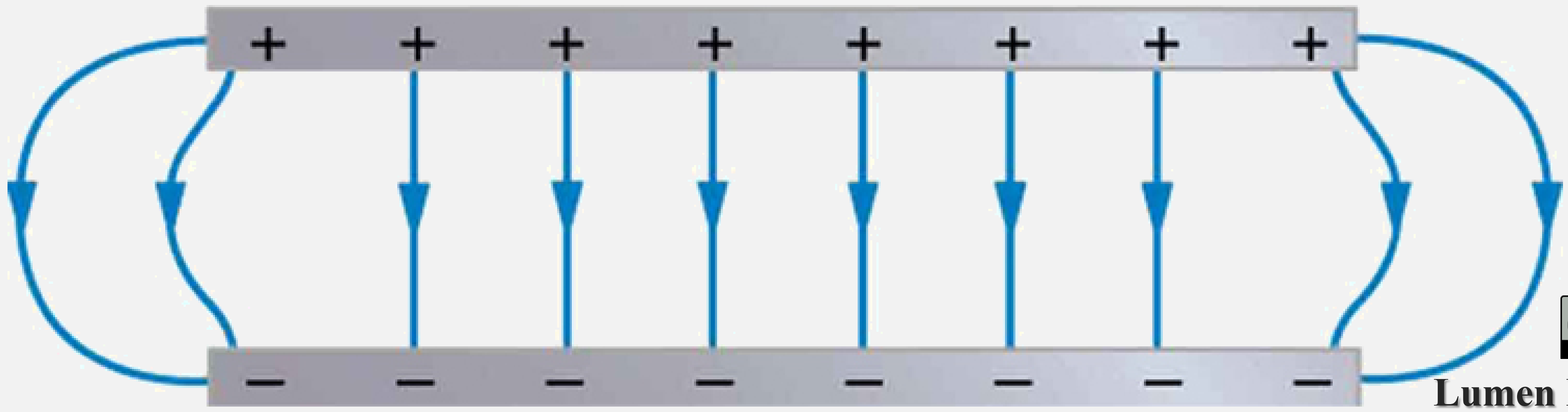
  $$\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot\left(\hat{x}V_x(x,y,z,t) + \hat{y}V_y(x,y,z,t) + \hat{z}V_z(x,y,z,t)\right) =$$

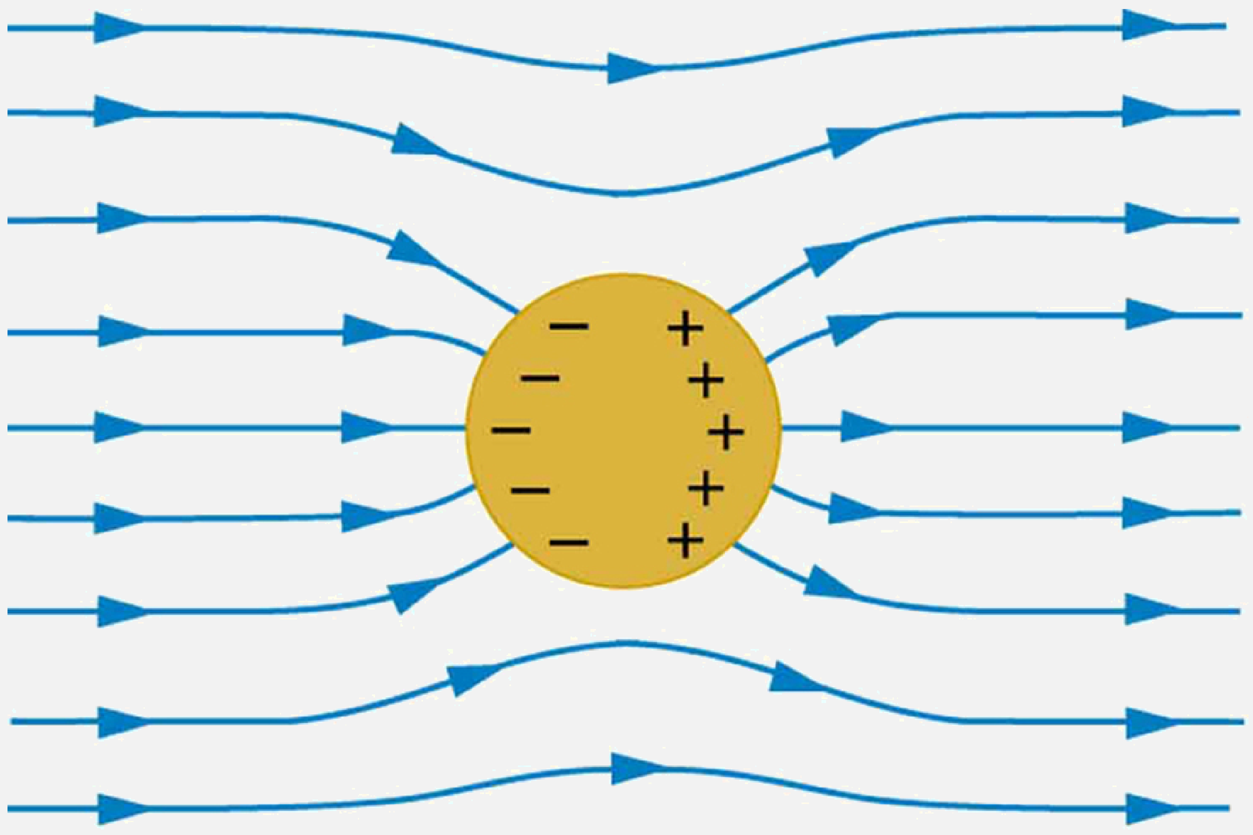
  $$\nabla\cdot\bar{V}(x,y,z,t)$$

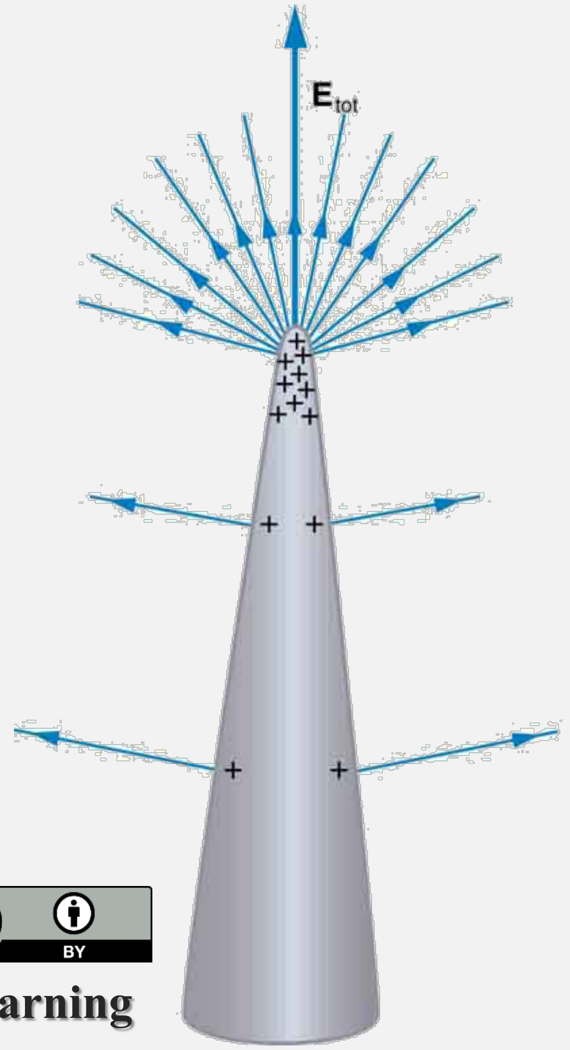
Sources and Sinks of Electric Field Lines



https://demo.webassign.net/ebooks/cj6demo/pc/c18/read/main/c18x18_7_ref.htm







https://courses.lumenlearning.com/physics/chapter/18-7-conductors-and-electric-fields-in-static-equilibrium/

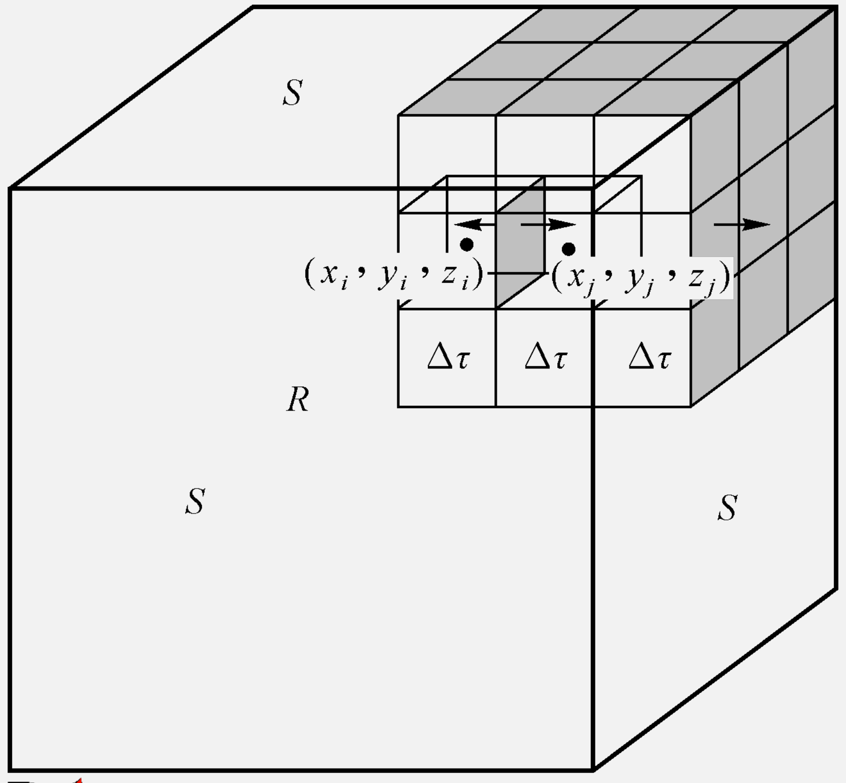
Divergence Theorem and Gauss's Law in Differential Form

- $\oint_S \bar{D}(x,y,z,t)\cdot\hat{n}da = \int_R \nabla\cdot\bar{D}(x,y,z,t)d\tau$
- $Q(t) = \int_R \rho_e(x,y,z,t)d\tau = \oint_S \bar{D}(x,y,z,t)\cdot\hat{n}da =$

$$\iint_{S_{yz}} D_x(x,y,z,t)n_x dydz + \iint_{S_{zx}} D_y(x,y,z,t)n_y dzdx$$

$$+\iint_{S_{xy}} D_z(x,y,z,t)n_z dxdy = \int_R \nabla\cdot\bar{D}(x,y,z,t)d\tau$$

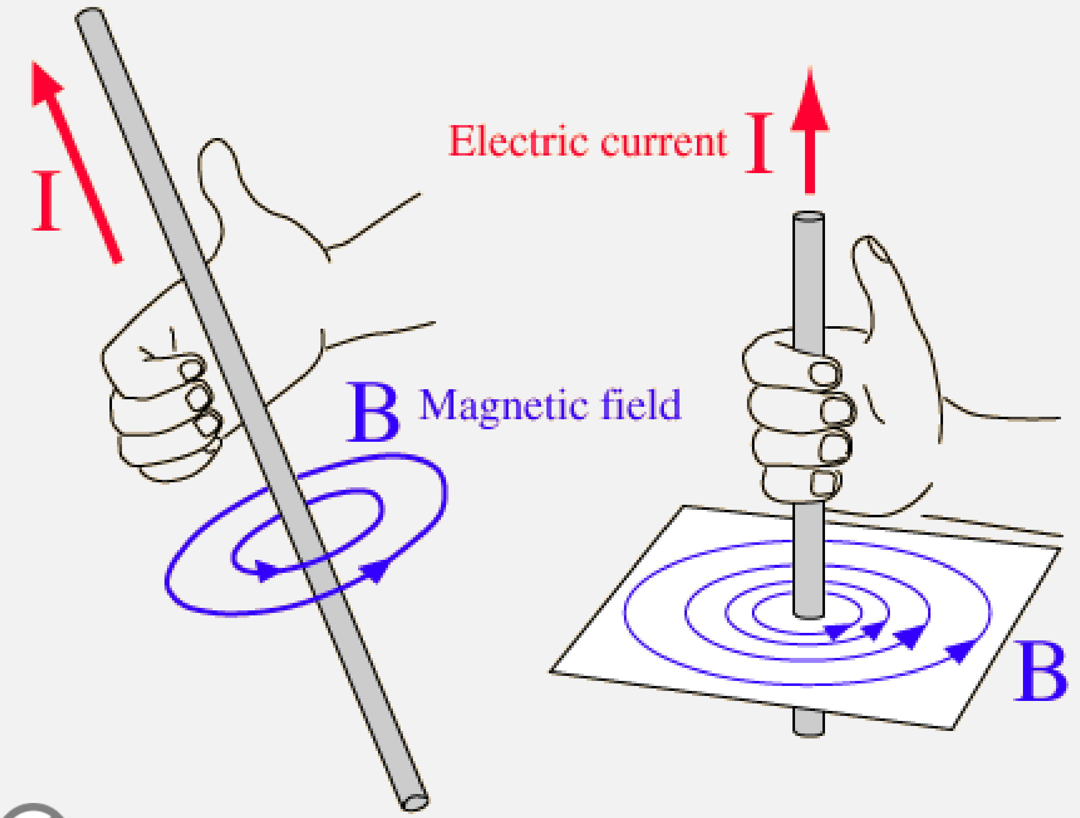
Let $R = \Delta\tau \to 0, \nabla\cdot\bar{D}(x,y,z,t) = \rho_e(x,y,z,t)$



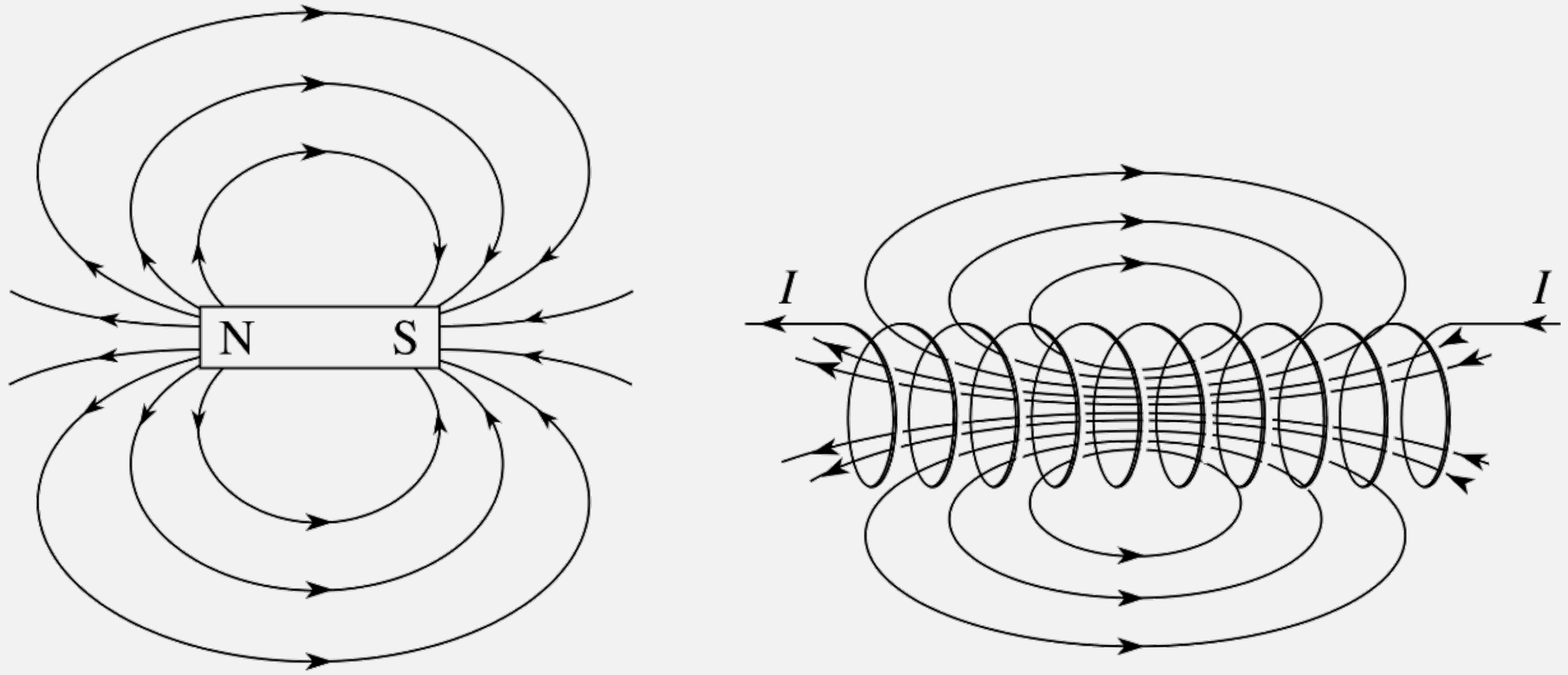
鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.

Gauss's Law for Magnetic Flux Density

- $\oint_S \bar{B}(\bar{r},t) \cdot \hat{n} da = 0$
- $\nabla \cdot \bar{B}(\bar{r},t) = \frac{\partial B_x(\bar{r},t)}{\partial x} + \frac{\partial B_y(\bar{r},t)}{\partial y} + \frac{\partial B_z(\bar{r},t)}{\partial z} = 0$



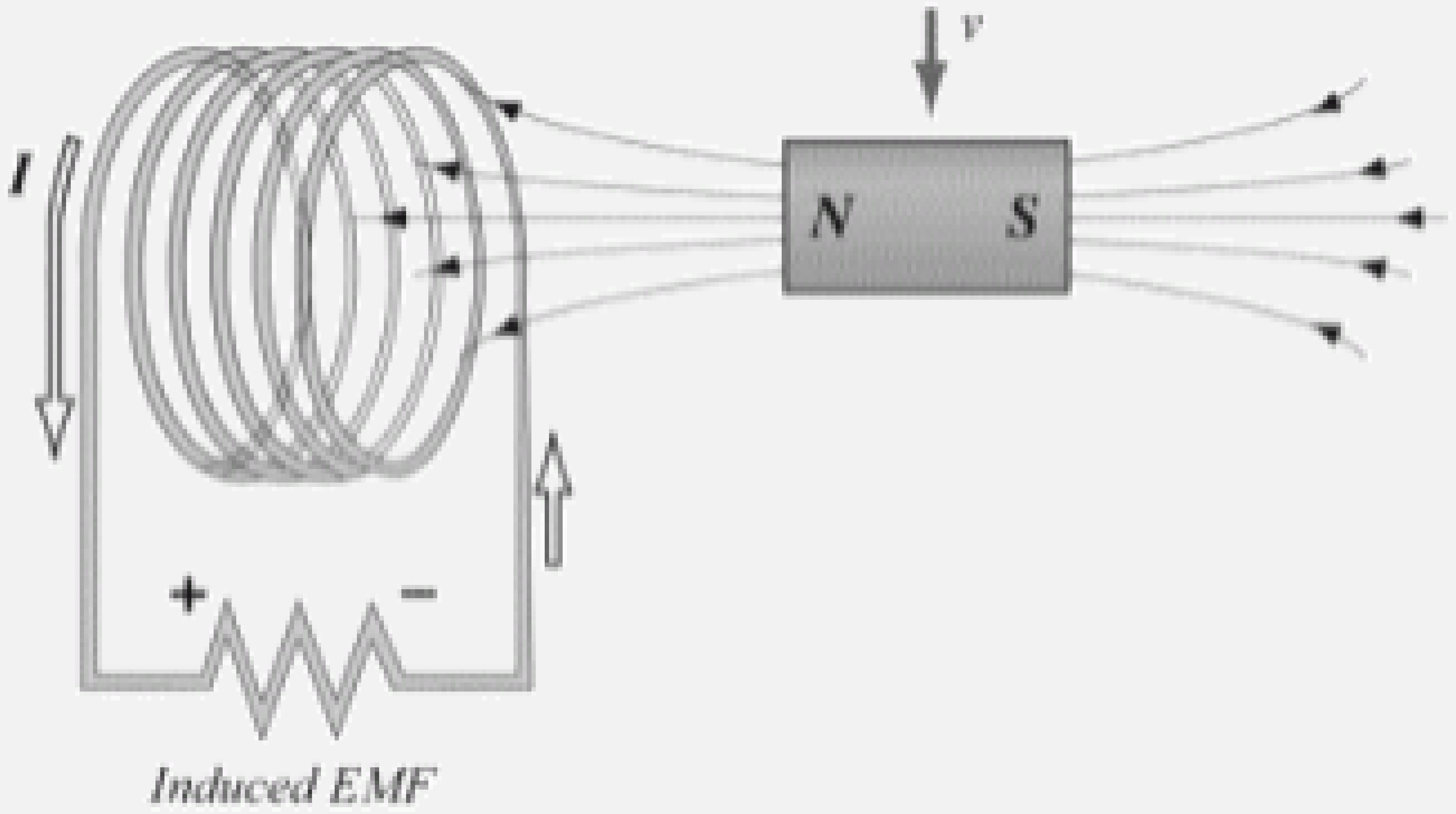
WIKIMEDIA COMMONS https://commons.wikimedia.org/wiki/File:V-1_right_hand_thumb_rule.gif



CBSE SCIENCE https://cbsescience.in/2017/09/27/magnetic-field-through-a-current-carrying-conductor/

Faraday's Law

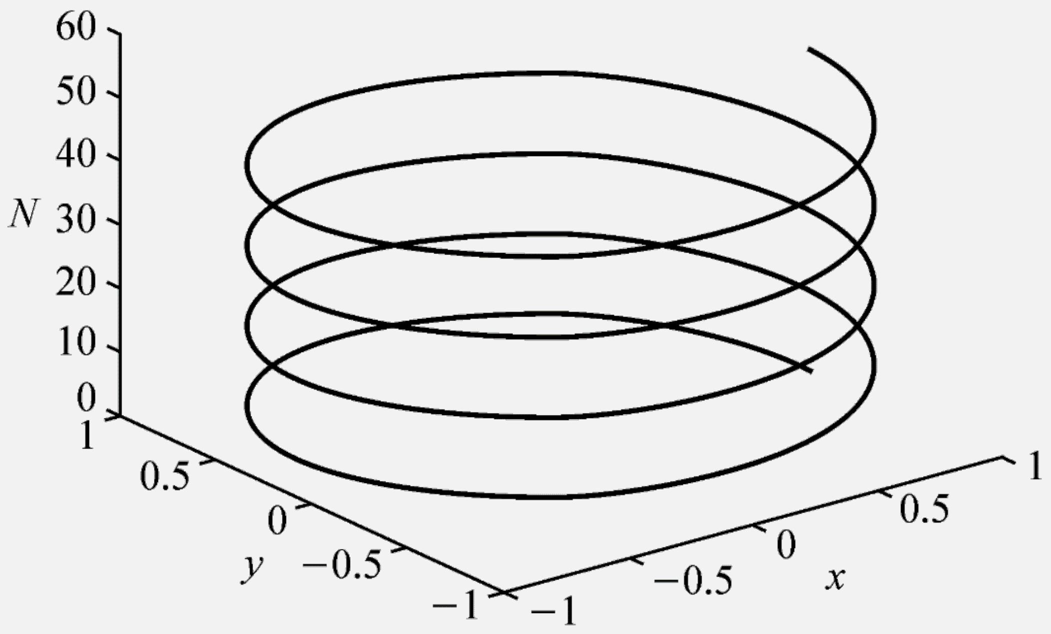
- $\varepsilon = -\frac{\Delta\Phi_M}{\Delta t}$



Suffolk University https://sites.suffolk.edu/kdshepard/2013/03/07/faradays-law/379a/

Line and Line Integral

- Line $\bar{r} = \bar{r}(\sigma) = \hat{x}x(\sigma) + \hat{y}y(\sigma) + \hat{z}z(\sigma)$
- Example
- $\bar{r}(\sigma) = \hat{x}\cos\sigma + \hat{y}\sin\sigma + \hat{z}\,2\,\sigma, 0 \le \sigma \le 8\pi$
- $\hat{t} = (\hat{x}dx + \hat{y}dy + \hat{z}dz)/d\ell$
- $d\ell = \sqrt{(dx)^2 + (dy)^2 + (dz)^2} = \sqrt{\left(\frac{dx}{d\sigma}\right)^2 + \left(\frac{dy}{d\sigma}\right)^2 + \left(\frac{dz}{d\sigma}\right)^2}\, d\sigma$
- Line integral
- $\int_C \bar{F}(r,t) \cdot \hat{t} d\ell =$
  $\int_{C_\sigma} \bar{F}(r,t) \cdot \hat{t} \sqrt{\left(\frac{dx}{d\sigma}\right)^2 + \left(\frac{dy}{d\sigma}\right)^2 + \left(\frac{dz}{d\sigma}\right)^2}\, d\sigma =$
  $\int_{C_x} F_x(r,t)\, dx + \int_{C_y} F_y(r,t)\, dy + \int_{C_z} F_z(r,t)\, dz$



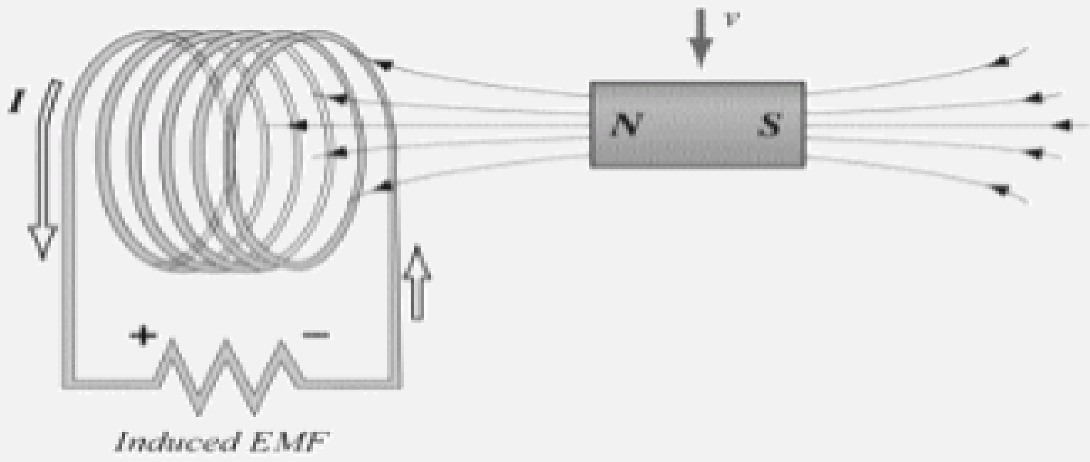
鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.

Electric Field Strength and Faraday's Law

- $\bar{E}(\bar{r},t) = \bar{F}(\bar{r},t)/q$
- $\varepsilon = -\frac{\Delta\Phi_M}{\Delta t}$
- $\oint_C \bar{E}(\bar{r},t)\cdot\hat{t}d\ell = -\frac{\partial}{\partial t}\int_S \bar{B}(\bar{r},t)\cdot\hat{n}da$
- $\int_{C_x} E_x(\bar{r},t)\,dx + \int_{C_y} E_y(\bar{r},t)\,dy + \int_{C_z} E_z(\bar{r},t)\,dz =$

$$-\frac{\partial}{\partial t}\iint_{S_{yz}} B_x(\bar{r},t)n_x dydz - \frac{\partial}{\partial t}\iint_{S_{zx}} B_y(\bar{r},t)n_y dzdx$$

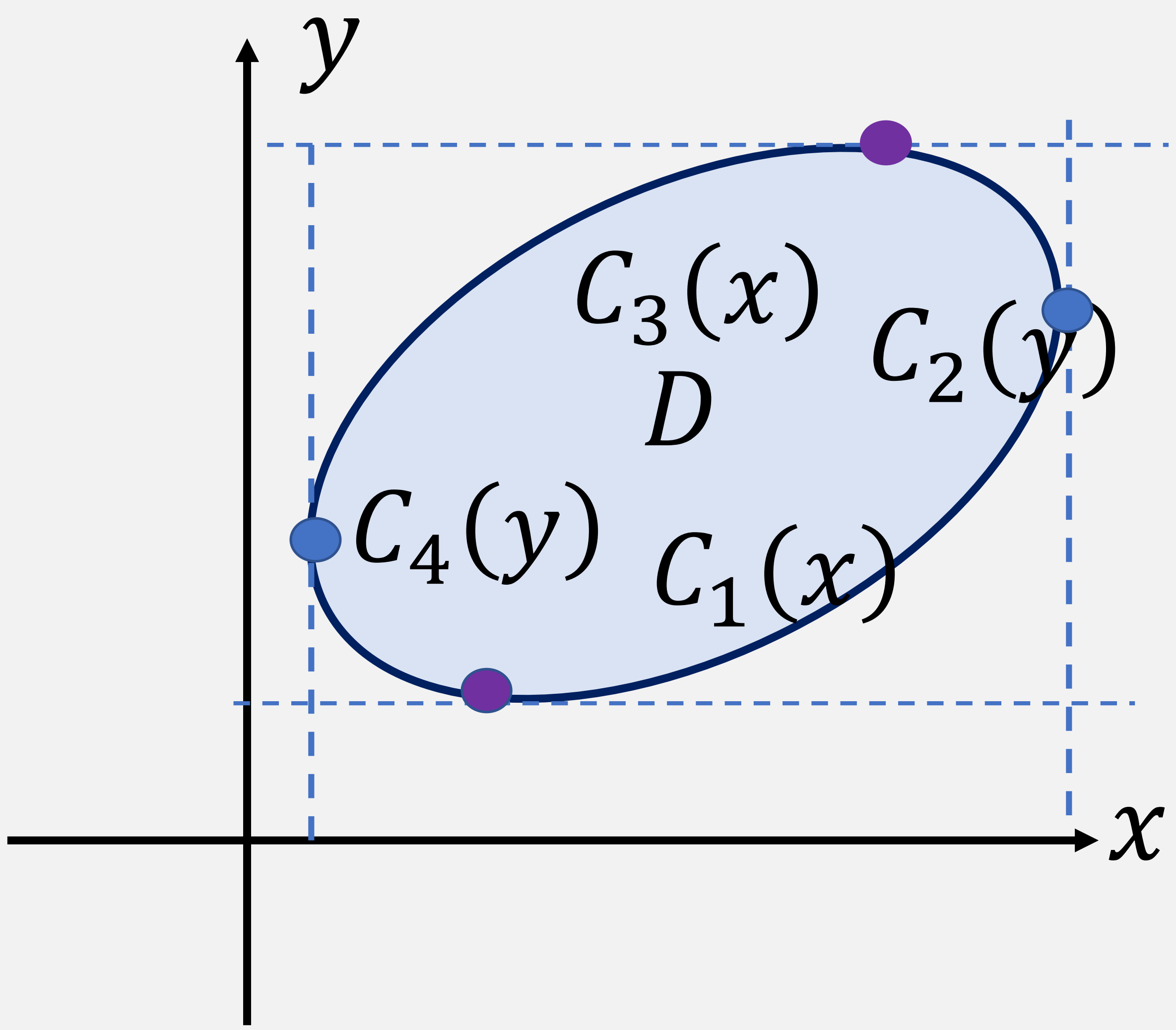
$$-\frac{\partial}{\partial t}\iint_{S_{xy}} B_z(\bar{r},t)n_z dxdy$$



Suffolk University
https://sites.suffolk.edu/kdshepard/2013/03/07/faradays-law/379a/

Green's Theorem

- $\oint_C \bar{F}(x,y)\cdot\hat{t}d\ell = \int_C F_x(x,y)dx + \int_C F_y(x,y)dy$
- $\int_{C_x} F_x(x,y)dx = -\int_a^b \int_{C_1(x)}^{C_3(x)} \frac{\partial}{\partial y} F_x(x,y)dydx$
- $\int_{C_y} F_y(x,y)dy = \int_c^d \int_{C_4(y)}^{C_2(y)} \frac{\partial}{\partial x} F_y(x,y)\,dxdy =$
- $\int_c^d \int_{C_2(y)}^{C_4(y)} \frac{\partial}{\partial x} F_y(x,y)\,dxdy$
- $\oint_C \bar{F}(x,y)\cdot\hat{t}d\ell =$
- $\int_{C_x} F_x(x,y)dx + \int_{C_y} F_y(x,y)dy =$
- $\int_D \left\{\frac{\partial}{\partial x} F_y(x,y) - \frac{\partial}{\partial y} F_x(x,y)\right\} dxdy$



Stokes Theorem

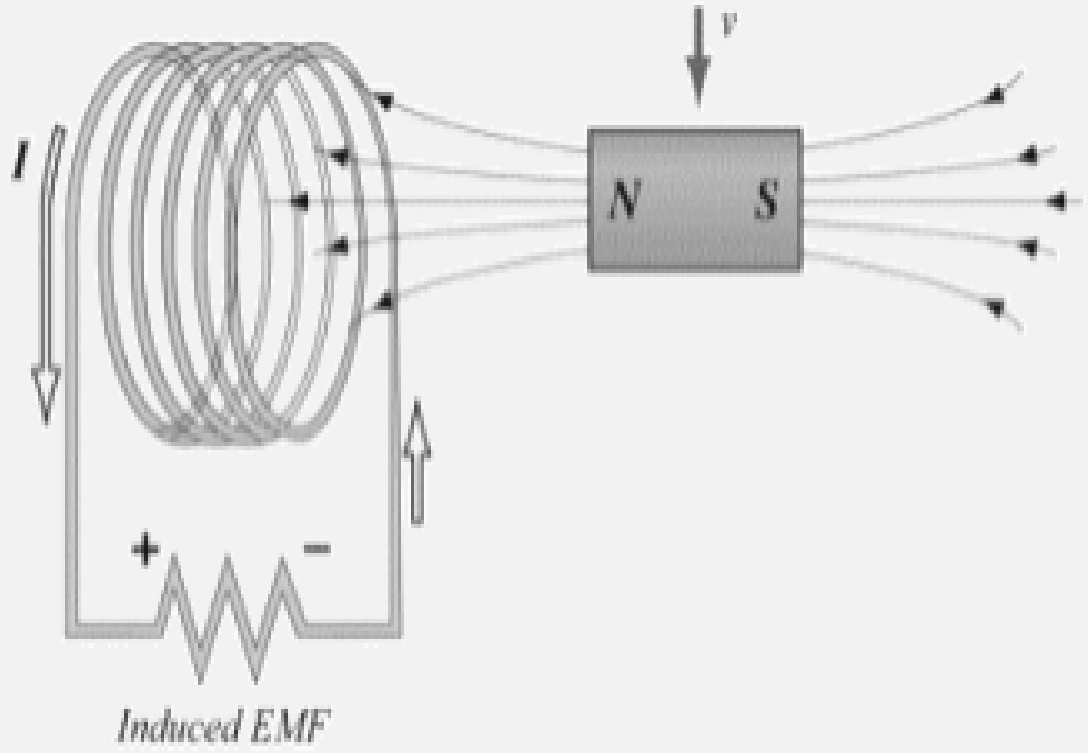
- $\int_{C_x} F_x(\bar{r},t)\,dx + \int_{C_y} F_y(\bar{r},t)\,dy + \int_{C_z} F_z(\bar{r},t)\,dz =$
- $\iint_{S_{yz}} \left\{\frac{\partial}{\partial y} F_z(\bar{r},t) - \frac{\partial}{\partial z} F_y(\bar{r},t)\right\} n_x dydz + \iint_{S_{zx}} \left\{\frac{\partial}{\partial z} F_x(\bar{r},t) - \frac{\partial}{\partial x} F_z(\bar{r},t)\right\} n_y dzdx + \iint_{S_{xy}} \left\{\frac{\partial}{\partial x} F_y(\bar{r},t) - \frac{\partial}{\partial y} F_x(\bar{r},t)\right\} n_z dxdy$
- $\oint_C \bar{F}(\bar{r},t)\cdot\hat{t}d\ell = \int_S curl\,\bar{F}(\bar{r},t)\cdot\hat{n}da,$

curl Operator

- $\oint_C \bar{F}(\bar{r},t)\cdot\hat{t}d\ell = \int_S curl\,\bar{F}(\bar{r},t)\cdot\hat{n}da,$
- $curl\,\bar{F}(\bar{r},t)\cdot\hat{n} = \lim_{\Delta a\to 0}\frac{\oint_C \bar{F}(\bar{r},t)\cdot\hat{t}d\ell}{\Delta a}$
- $curl\,\bar{F}(\bar{r},t) = \hat{x}\left\{\frac{\partial}{\partial y}F_z(\bar{r},t) - \frac{\partial}{\partial z}F_y(\bar{r},t)\right\} + \hat{y}\left\{\frac{\partial}{\partial z}F_x(\bar{r},t) - \frac{\partial}{\partial x}F_z(\bar{r},t)\right\} + \hat{z}\left\{\frac{\partial}{\partial x}F_y(\bar{r},t) - \frac{\partial}{\partial y}F_x(\bar{r},t)\right\} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x(\bar{r},t) & F_y(\bar{r},t) & F_z(\bar{r},t) \end{vmatrix} = \nabla\times\bar{F}(\bar{r},t)$

Differential Form of Faraday's Law

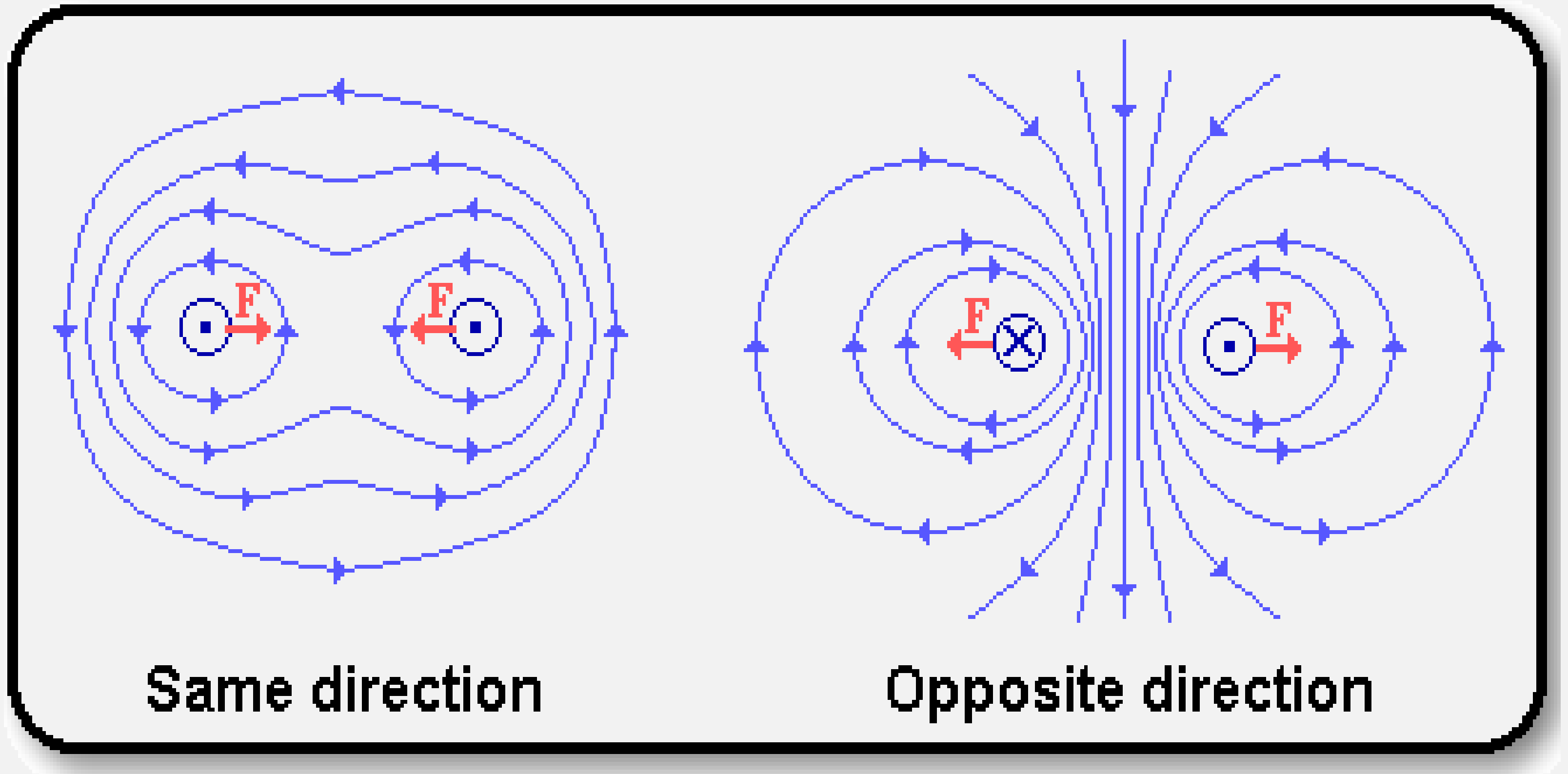
- $\oint_C \bar{E}(\bar{r},t)\cdot\hat{t}d\ell = -\frac{\partial}{\partial t}\int_S \bar{B}(\bar{r},t)\cdot\hat{n}da$
- $\int_S \nabla\times\bar{E}(\bar{r},t)\cdot\hat{n}da = -\frac{\partial}{\partial t}\int_S \bar{B}(\bar{r},t)\cdot\hat{n}da$
- Let $s = \Delta s \to 0$
- $\hat{x}\cdot\nabla\times\bar{E}(\bar{r},t)\,n_x\Delta s = -\frac{\partial}{\partial t}\hat{x}\cdot\frac{\partial}{\partial t}\bar{B}(\bar{r},t)n_x\Delta s$
- $\hat{y}\cdot\nabla\times\bar{E}(\bar{r},t)\,n_y\Delta s = -\frac{\partial}{\partial t}\hat{y}\cdot\frac{\partial}{\partial t}\bar{B}(\bar{r},t)n_y\Delta s$
- $\hat{z}\cdot\nabla\times\bar{E}(\bar{r},t)\,n_z\Delta s = -\frac{\partial}{\partial t}\hat{z}\cdot\frac{\partial}{\partial t}\bar{B}(\bar{r},t)n_z\Delta s$
- $\nabla\times\bar{E}(\bar{r},t) = -\frac{\partial}{\partial t}\bar{B}(\bar{r},t)$



Suffolk University
https://sites.suffolk.edu/kdshepard/2013/03/07/faradays-law/379a/

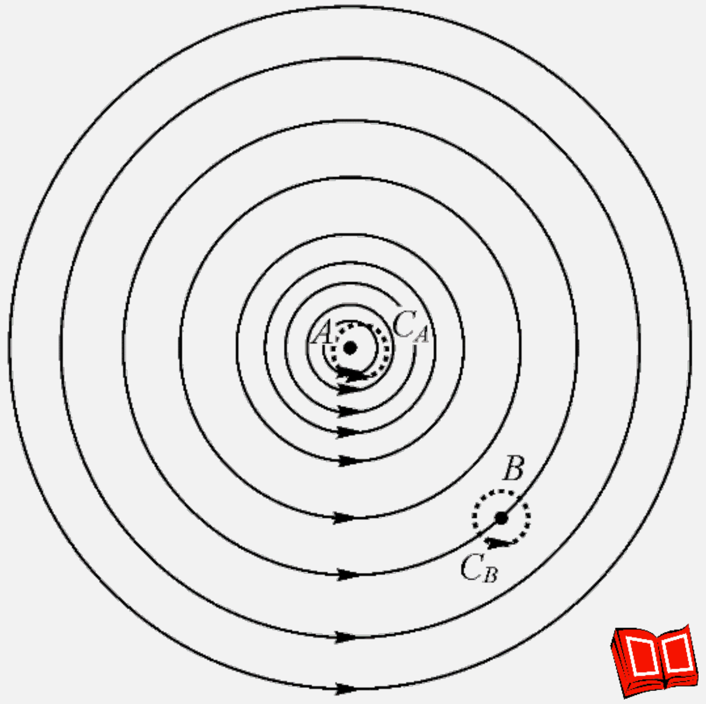
Ampère’s Circuital Law

- $\oint_C \bar{H}(\bar{r},t)\cdot\hat{t}d\ell = \int_S \bar{J}(\bar{r},t)\cdot\hat{n}da$
- $curl\ \bar{H}(\bar{r},t) = \nabla\times\bar{H}(\bar{r},t) = \bar{J}(\bar{r},t)$

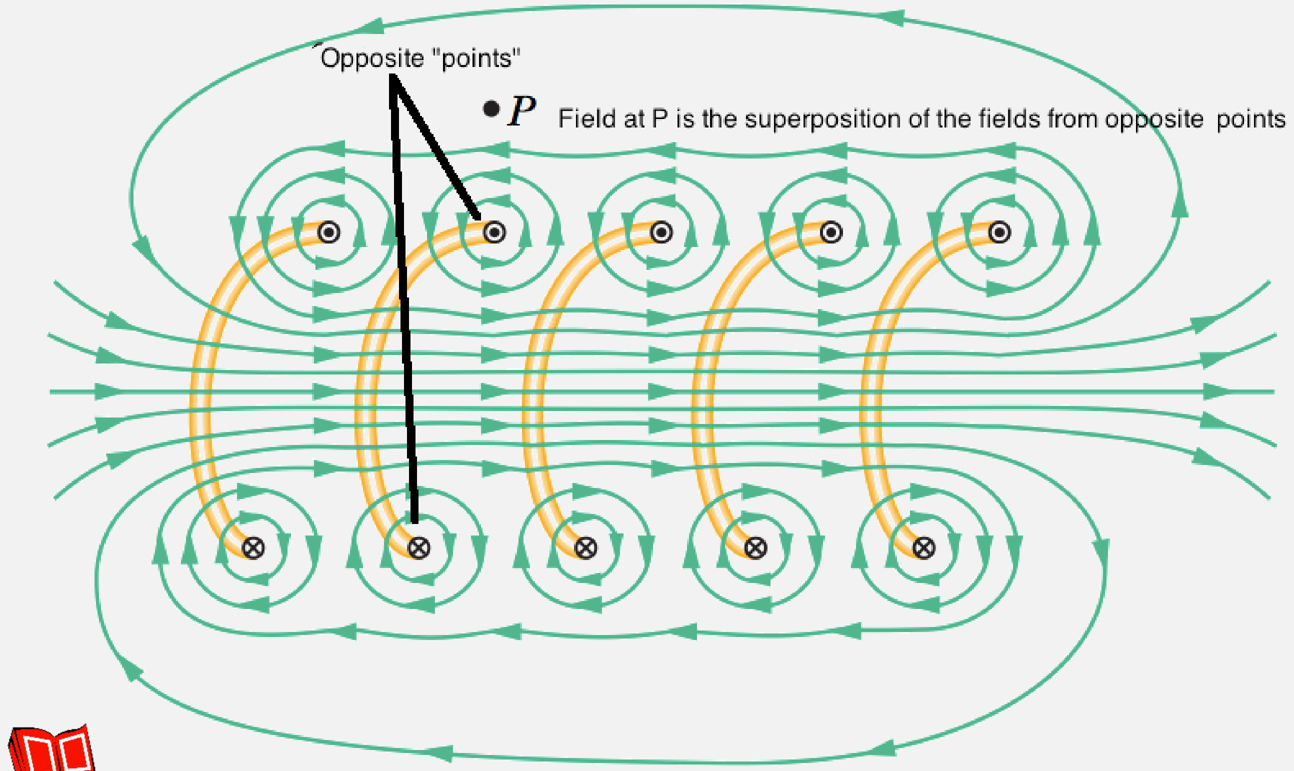


Magnetic field line due to two current carrying wires

goprep https://goprep.co/what-is-the-magnetic-field-midway-between-two-parallel-i-1nlo17



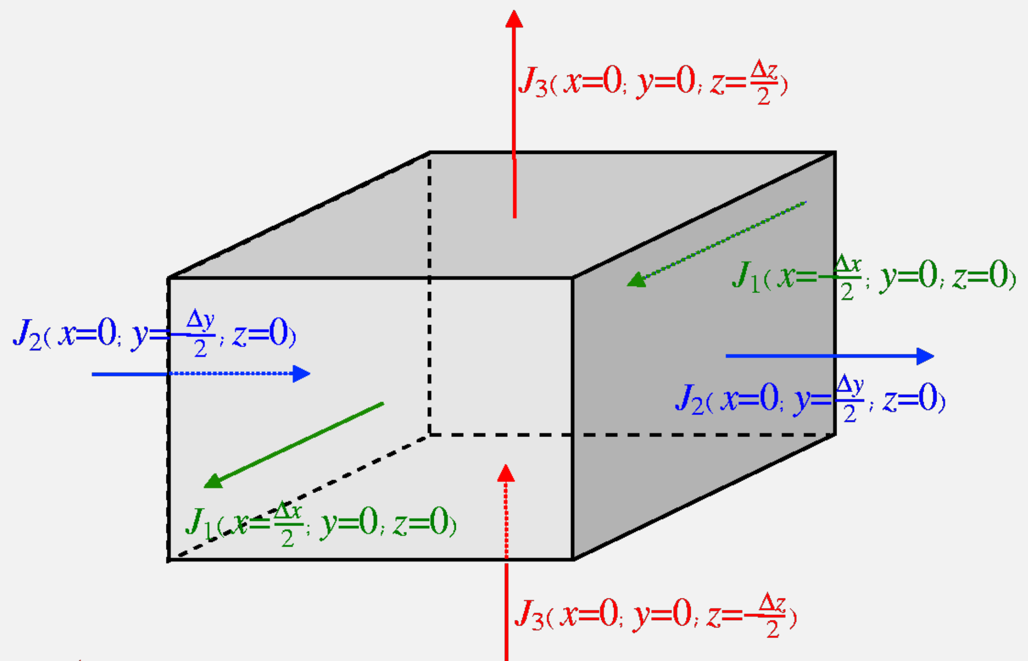
鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.



Stock Exchange
https://physics.stackexchange.com/questions/102312/what-are-the-limits-of-validity-for-the-magnetic-field-of-a-solenoid

Conservation of Charges

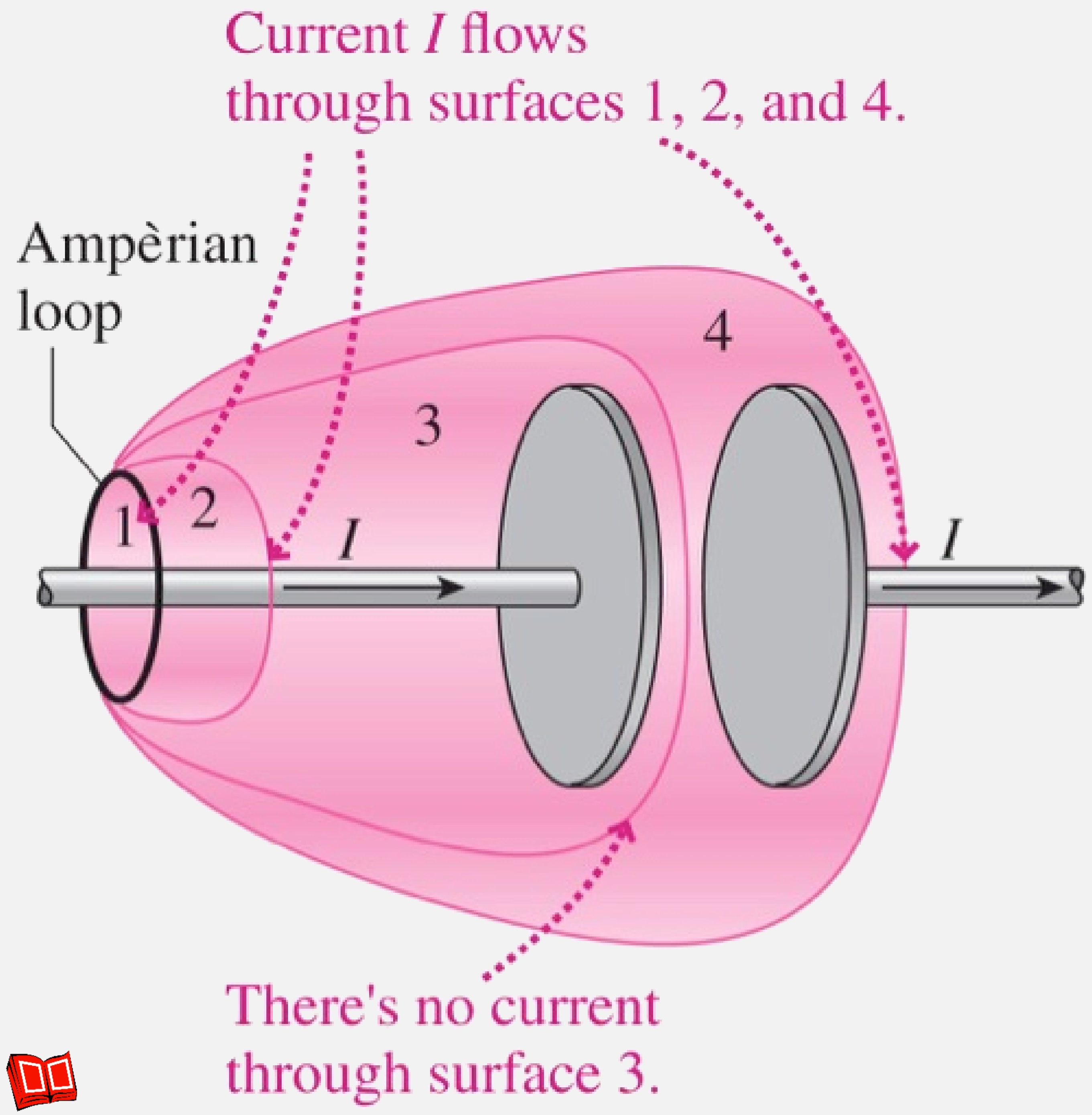
- $\oint_S \bar{J}(\bar{r},t)\cdot\hat{n}da + \frac{\partial}{\partial t}Q(\mathrm{t}) = 0$
- $\int_R \nabla\cdot\bar{J}(\bar{r},t)d\tau + \frac{\partial}{\partial t}\int_R \rho_e(\bar{r},t)d\tau = 0$
- $\nabla\cdot\bar{J}(\bar{r},t) + \frac{\partial}{\partial t}\rho_e(\bar{r},t) = 0$



Massachusetts Institute of Technology
http://pruffle.mit.edu/3.016-2005/Lecture_16_web/node2.html

Inconsistency of Ampere's Circuital Law

- $\oint_C \bar{H}(\bar{r},t)\cdot\hat{t}d\ell = \int_S \bar{J}(\bar{r},t)\cdot\hat{n}da$
- $\nabla\times\bar{H}(\bar{r},t) = \bar{J}(\bar{r},t)$
- $\nabla\cdot\nabla\times\bar{H}(\bar{r},t) = \nabla\cdot\bar{J}(\bar{r},t)$
- $\nabla\cdot\nabla\times\bar{H}(\bar{r},t) \equiv 0$
- $\nabla\cdot\bar{J}(\bar{r},t) = 0 \neq -\frac{\partial}{\partial t}\rho_e(\bar{r},t)$



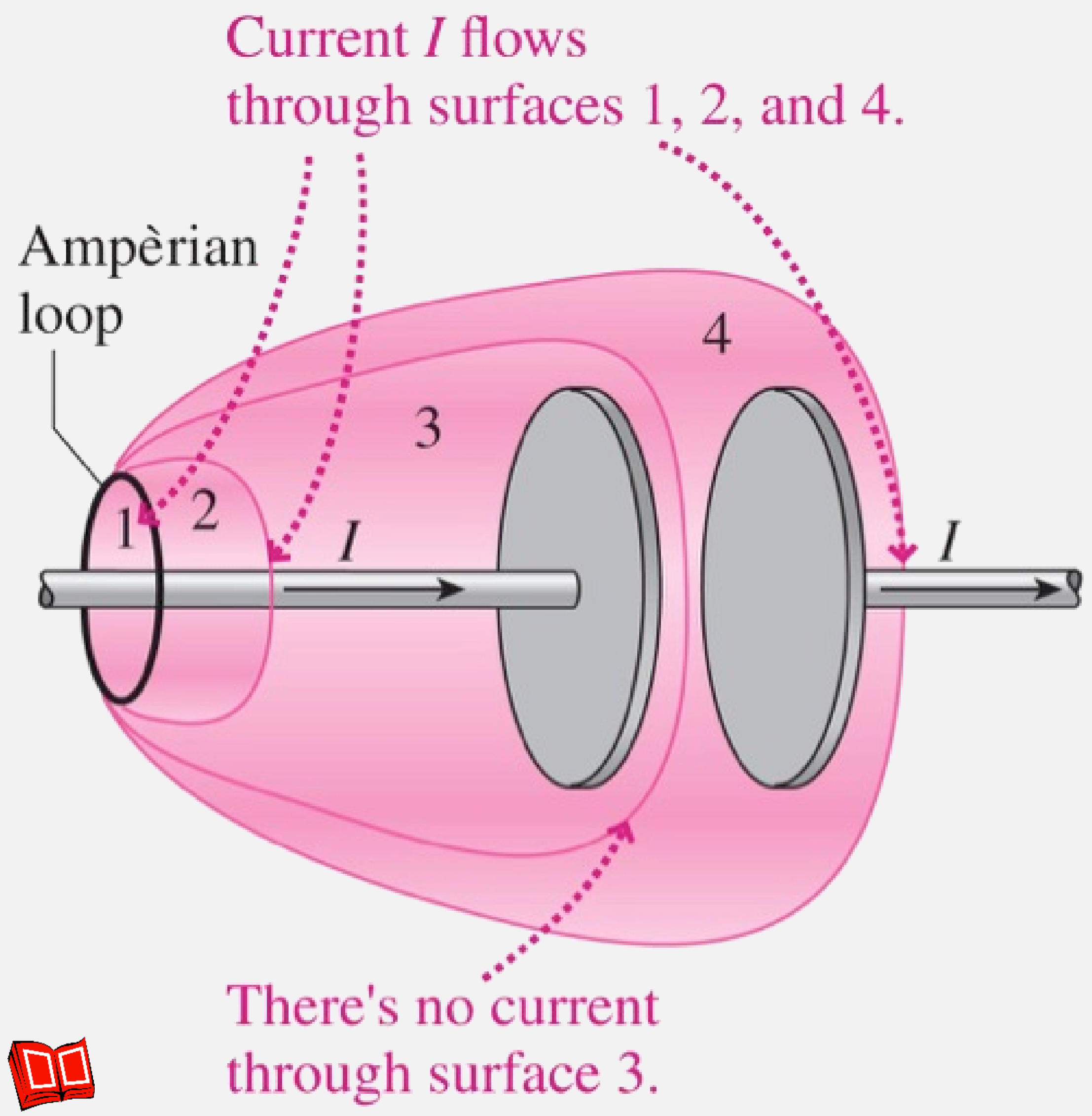
Physics Department, University of Louisville
https://www.physics.louisville.edu/cldavis/phys299/notes/mag_displacement.html

Maxwell's Displacement Current Density

- $\oint_C \; \bar{H}(\bar{r},t)\cdot\hat{t}\,d\ell = \int_S \left\{\bar{J}(\bar{r},t) + \frac{\partial}{\partial t}\bar{D}(\bar{r},t)\right\}\cdot\hat{n}\,da$
- $\nabla\times\bar{H}(\bar{r},t) = \bar{J}(\bar{r},t) + \frac{\partial}{\partial t}\bar{D}(\bar{r},t)$
- $\nabla\cdot\nabla\times\bar{H}(\bar{r},t) =$

  $0 = \nabla\cdot\bar{J}(\bar{r},t) + \frac{\partial}{\partial t}\nabla\cdot\bar{D}(\bar{r},t) =$

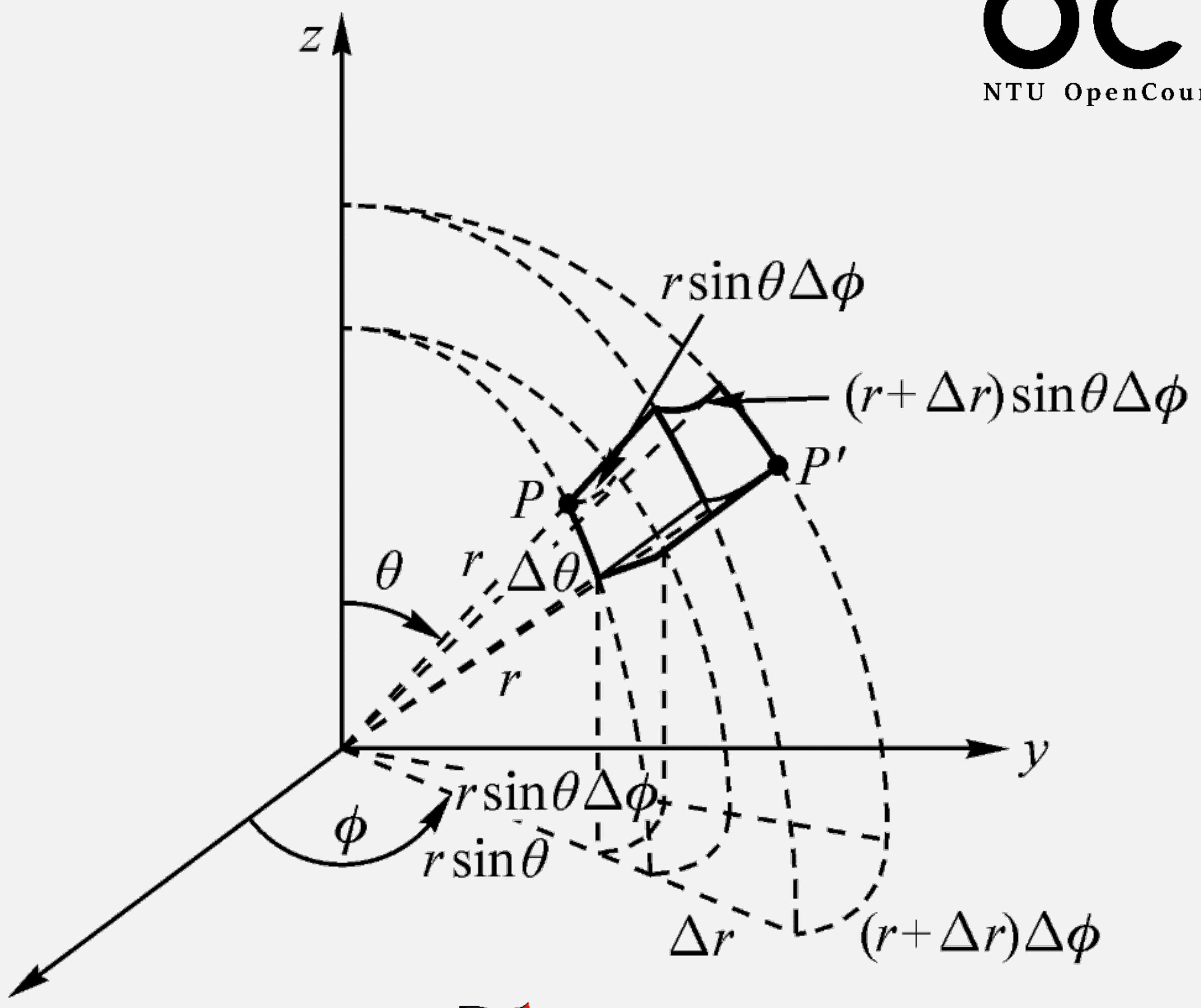
  $\nabla\cdot\bar{J}(\bar{r},t) + \frac{\partial}{\partial t}\rho_e(\bar{r},t)$



Physics Department, University of Louisville
https://www.physics.louisville.edu/cldavis/phys299/notes/mag_displacement.html

Permittivity of Free Space

- $\bar{D}(\bar{r},t) = \epsilon_0 \bar{E}(\bar{r},t)$
- $\oint_S \bar{D} \cdot \hat{n} da = q$
- Coulomb's law: $\bar{E} = K\frac{q}{r^2}$
- Spherical surface:
$\bar{r}(\theta,\phi) = \hat{x}\, r\sin\theta\cos\phi + \hat{y} r\sin\theta\sin\phi + \hat{z}\, r\cos\theta$
- Differential solid angle: $d\Omega = \sin\theta \; d\theta d\phi$
- $q = \int_0^{2\pi}\int_0^{\pi} \epsilon_0 K\frac{q}{r^2} r^2 \sin\theta d\theta d\phi = 4\pi\epsilon_0 K q$
- $K = \frac{1}{4\pi\epsilon_0}$



鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.

Permeability of Free Space

- $\bar{B}(\bar{r}, t) = \mu_0 \bar{H}(\bar{r}, t)$

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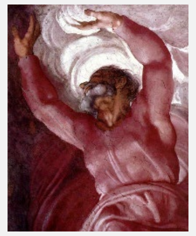
2			WIKIMEDIA COMMONS / File:Dividing Light from Darkness.jpg / Michelangelo (1475-1564) / 1509 / Musei Vaticani， https://commons.wikimedia.org/wiki/File:Dividing_Light_from_Darkness.jpg， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
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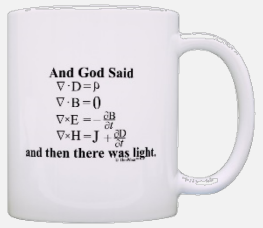
3			amazon / Funny Math Gift God Said Maxwell's Equations There Was Light Gift Coffee Mug Tea Cup White / ThisWear，https://www.amazon.com/Funny-Maxwells-Equations-There-Coffee/dp/B017OE2KTI/ref=sr_1_1?crid=P8CII24NYLH&keywords=maxwell%2Bequation%2Bmug&qid=1566367884&s=gateway&sprefix=Maxwell%2Beq%2Caps%2C358&sr=8-1&th=1， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
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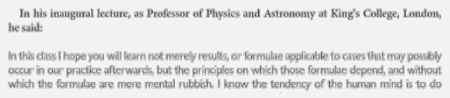
4	 In his inaugural lecture, as Professor of Physics and Astronomy at King's College, London, he said: In this class I hope you will learn not merely results, or formulae applicable to cases that may possibly occur in our practice afterwards, but the principles on which those formulae depend, and without which the formulae are mere mental rubbish. I know the tendency of the human mind is to do		Google Books / James Clerk Maxwell: Perspectives on his Life and Work / edited by Raymond Flood, Mark McCartney, Andrew Whitaker，https://books.google.com.tw/books?id=IMl7AgAAQBAJ&pg=PA8&lpg=PA8&dq=in+this+class+i+hope+you+will+learn+not+merely+results+maxwell&source=bl&ots=UX09wsz5Ux&sig=ACfU3U2-n1ZHGORdlF8DB8ct9LBdYLl-CA&hl=en&sa=X&ved=2ahUKEwjDyOvg-cn1AhUyLqYKHYQgCVcQ6AF6BAgCEAM#v=onepage&q&f=false，本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
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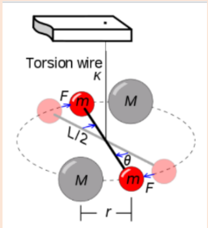
6			WIKIMEDIA COMMONS / File:Cavendish Torsion Balance Diagram.svg / 31 August 2007 / Chris Burks (Chetvorno)，https://commons.wikimedia.org/wiki/File:Cavendish_Torsion_Balance_Diagram.svg，Public domain，2022/01/27 瀏覽。

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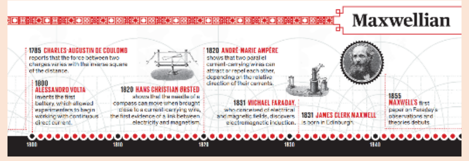
7			IEEE Xplore / The Long Road to Maxwell‘s Equations / by James C. Rautio，https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=6964925，本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。

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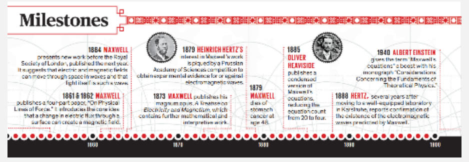
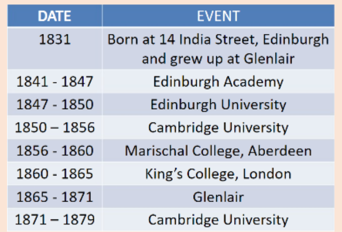
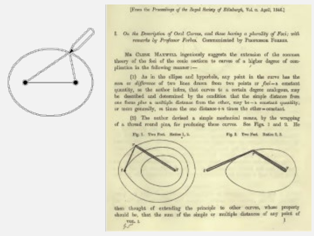
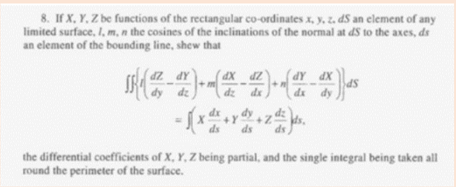
7			IEEE Xplore / The Long Road to Maxwell's Equations / by James C. Rautio， https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=6964925， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
10	 DATE | EVENT 1831 | Born at 14 India Street, Edinburgh and grew up at Glenlair 1841 - 1847 | Edinburgh Academy 1847 - 1850 | Edinburgh University 1850 – 1856 | Cambridge University 1856 - 1860 | Marischal College, Aberdeen 1860 - 1865 | King's College, London 1865 - 1871 | Glenlair 1871 – 1879 | Cambridge University		Slide Player / James Clerk Maxwell 1831-1879 / Raymond Flood / P.2， https://slideplayer.com/slide/3796167/， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
11			Slide Player / James Clerk Maxwell 1831-1879 / Raymond Flood / P.5， https://slideplayer.com/slide/3796167/， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
12			Slide Player / James Clerk Maxwell 1831-1879 / Raymond Flood / P.8， https://slideplayer.com/slide/3796167/， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。

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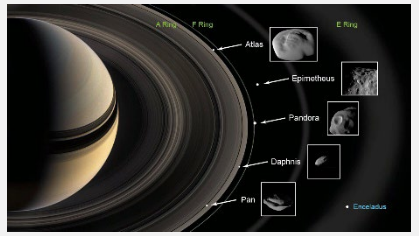
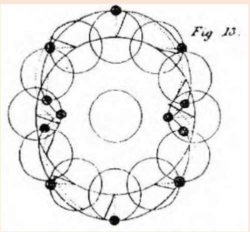
14			NASA Science / Cassini Finds Saturn‘s Rings Coat Tiny Moons / Among the Rings / Credit: NASA-JPL/Caltech，https://solarsystem.nasa.gov/news/886/cassini-finds-saturns-rings-coat-tiny-moons/，本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
14			郭奕玲, 沈慧君編, 電磁場理論的奠基人—麥克斯韋, , 凡異出版社, 1999, 圖 1-17, p. 3.，本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。

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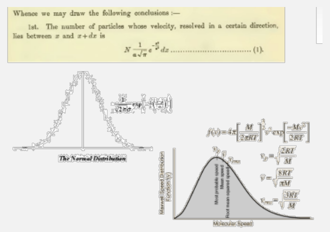
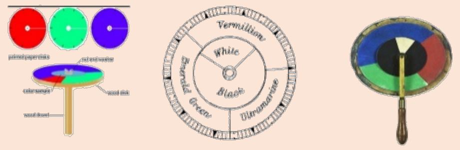
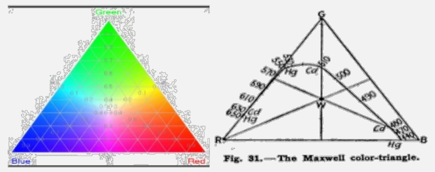
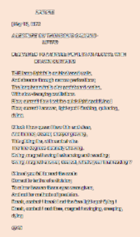
14			Slide Player / James Clerk Maxwell 1831-1879 / Raymond Flood / P.28， https://slideplayer.com/slide/3796167/， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
14			SPIE. Digital Library / Maxwell, color vision, and the color triangle / 30 August 2019 / Vasudevan Lakshminarayanan / The color top.， https://www.spiedigitallibrary.org/conference-proceedings-of-spie/11099/110990A/Maxwell-color-vision-and-the-color-triangle/10.1117/12.2529364.full?SSO=1， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
14			SPIE. Digital Library / Maxwell, color vision, and the color triangle / 30 August 2019 / Vasudevan Lakshminarayanan / The Maxwell Triangle， https://www.spiedigitallibrary.org/conference-proceedings-of-spie/11099/110990A/Maxwell-color-vision-and-the-color-triangle/10.1117/12.2529364.full?SSO=1， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
15			Scottish Science Hall of Fame / James Clerk Maxwell (1831-1879) / Lecture on Thomson's galvanometer : delivered to a single pupil in an alcove with drawn curtains， https://digital.nls.uk/scientists/archive/74629736， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。

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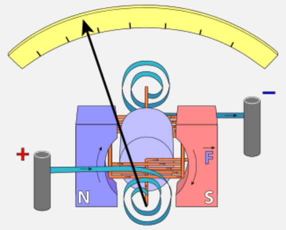
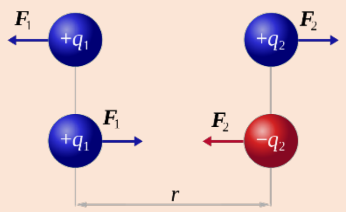
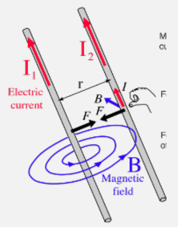
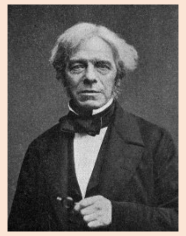
15		CC BY SA	WIKIMEDIA COMMONS / File:Galvanometer scheme.svg / 2 December 2011 / Fred the Oyster， https://commons.wikimedia.org/wiki/File:Galvanometer_scheme.svg， CC BY-SA 3.0，2022/01/27 瀏覽。
17		CC BY	WIKIMEDIA COMMONS / File:CoulombsLaw.svg / 24 January 2008 / User:Dna-Dennis， https://commons.wikimedia.org/wiki/File:CoulombsLaw.svg， CC BY 3.0，2022/01/27 瀏覽。
17			Georgia State University / Magnetic Force Between Wires， http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/wirfor.html， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
18			WIKIMEDIA COMMONS / File:Faraday-Millikan-Gale-1913.jpg / circa 1861 / Opposite p. 290 of Millikan and Gale's *Practical Physics* (1922)， https://commons.wikimedia.org/wiki/File:Faraday-Millikan-Gale-1913.jpg， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。

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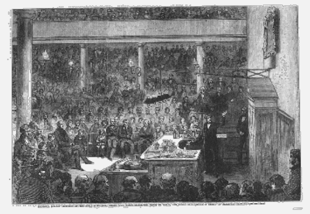
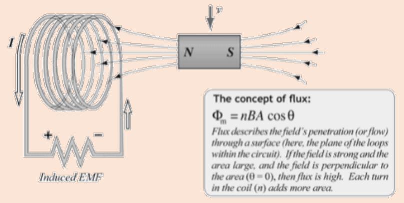
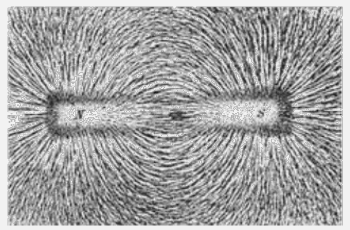
18			WIKIMEDIA COMMONS / File:Faraday lecture at Royal Institution 1856.jpg / H.B.H. from sketch by Alexander Blaikley / 16 February 1856，https://commons.wikimedia.org/wiki/File:Faraday_lecture_at_Royal_Institution_1856.jpg，本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
18、46、48、52			Suffolk University / Discovering The World / Faraday's Law，https://sites.suffolk.edu/kdshepard/2013/03/07/faradays-law/379a/，本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
18-19、22、35			Google Books / Practical Physics / Newton Henry Black，https://books.google.com.tw/books?id=BT8AAAAAYAAJ&pg=PA242&redir_esc=y#v=onepage&q&f=false，本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
19	… by line of magnetic force, …… be either in curved or straight lines.		Michael Faraday, Experimental Researches in Electricity, vol. III, London: Bernard Quaritch, 1855, ¶ 2149.，本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。

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21	By a physical analogy …… of quantities by operations on numbers.		James Clerk Maxwell, On Faraday's Lines of Force, Transactions of the Cambridge Philosophical Society, vol. x, part 1, 1855. p. 155.，本網站係以著作權法第 46、52、65 條合理使用本件作品。
22	"[T]his curve will indicate the direction of that …… on that account *a line of force* [Maxwell's emphasis]."		James Clerk Maxwell, On Faraday's Lines of Force, Transactions of the Cambridge Philosophical Society, vol. x, part 1, 1855. p. 158.，本網站係以著作權法第 46、52、65 條合理使用本件作品。
22	"[C]onsider these curves not merely lines, …… by the motion of the fluid in these tubes."		James Clerk Maxwell, On Faraday's Lines of Force, Transactions of the Cambridge Philosophical Society, vol. x, part 1, 1855. p. 159.，本網站係以著作權法第 46、52、65 條合理使用本件作品。
23	Three equations of Magnetic Force . . . . (B) " Electric Currents . . . . (C) " Electromotive Force . . . . (D) " Electric Elasticity . . . . (E) " Electric Resistance . . . . (F) " Total Currents . . . . (A) One equation of Free Electricity . . . . (G) " Continuity . . . . (H)		James Clerk Maxwell, "A dynamic theory of the electromagnetic field," Royal Society Transactions, vol. 155, 1865, pp. 459-512.，本網站係以著作權法第 46、52、65 條合理使用本件作品。

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23	For Electromagnetic Momentum . . . F G H " Magnetic Intensity . . . α β γ " Electromotive Force . . . P Q R " Current due to true conduction . . . p q r " Electric Displacement . . . f g h " Total Current (including variation of displacement) . . . p' q' r' " Quantity of free Electricity . . . e " Electric Potential . . . Ψ		James Clerk Maxwell, "A dynamic theory of the electromagnetic field," Royal Society Transactions, vol. 155, 1865, pp. 459-512.，本網站係以著作權法第 46、52、65 條合理使用本件作品。
24	$p'=p+\frac{df}{dt}$, $q'=q+\frac{dg}{dt}$, $r'=r+\frac{dh}{dt}$,		James Clerk Maxwell, "A dynamic theory of the electromagnetic field," Royal Society Transactions, vol. 155, 1865, pp. 459-512.，本網站係以著作權法第 46、52、65 條合理使用本件作品。
24	$\mu\alpha=\frac{dH}{dy}-\frac{dG}{dz}$, $\mu\beta=\frac{dF}{dz}-\frac{dH}{dx}$, $\mu\gamma=\frac{dG}{dx}-\frac{dF}{dy}$.		James Clerk Maxwell, "A dynamic theory of the electromagnetic field," Royal Society Transactions, vol. 155, 1865, pp. 459-512.，本網站係以著作權法第 46、52、65 條合理使用本件作品。
24	$\frac{d\gamma}{dy}-\frac{d\beta}{dz}=4\pi p'$. $\frac{d\alpha}{dz}-\frac{d\gamma}{dx}=4\pi q'$, $\frac{d\beta}{dx}-\frac{d\alpha}{dy}=4\pi r'$.		James Clerk Maxwell, "A dynamic theory of the electromagnetic field," Royal Society Transactions, vol. 155, 1865, pp. 459-512.，本網站係以著作權法第 46、52、65 條合理使用本件作品。

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25	$\left.\begin{aligned} P&=\mu\left(\gamma\frac{dy}{dt}-\beta\frac{dz}{dt}\right)-\frac{dF}{dt}-\frac{d\Psi}{dx}, \\ Q&=\mu\left(\alpha\frac{dz}{dt}-\gamma\frac{dx}{dt}\right)-\frac{dG}{dt}-\frac{d\Psi}{dy}, \\ R&=\mu\left(\beta\frac{dx}{dt}-\alpha\frac{dy}{dt}\right)-\frac{dH}{dt}-\frac{d\Psi}{dz}. \end{aligned}\right\}$		James Clerk Maxwell, “A dynamic theory of the electromagnetic field,” Royal Society Transactions, vol. 155, 1865, pp. 459-512.，本網站依著作權法第 46、52、65 條合理使用本件作品。
25	$\left.\begin{aligned} P&=kf, \\ Q&=kg, \\ R&=kh. \end{aligned}\right\}$		James Clerk Maxwell, “A dynamic theory of the electromagnetic field,” Royal Society Transactions, vol. 155, 1865, pp. 459-512.，本網站依著作權法第 46、52、65 條合理使用本件作品。
25	$\left.\begin{aligned} P&=-\varrho p, \\ Q&=-\varrho q, \\ R&=-\varrho r. \end{aligned}\right\}$		James Clerk Maxwell, “A dynamic theory of the electromagnetic field,” Royal Society Transactions, vol. 155, 1865, pp. 459-512.，本網站依著作權法第 46、52、65 條合理使用本件作品。
25	$e+\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}=0.$		James Clerk Maxwell, “A dynamic theory of the electromagnetic field,” Royal Society Transactions, vol. 155, 1865, pp. 459-512.，本網站依著作權法第 46、52、65 條合理使用本件作品。

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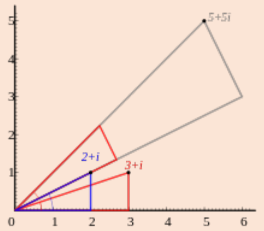
25	$\frac{de}{dt}+\frac{dp}{dx}+\frac{dq}{dy}+\frac{dr}{dz}=0$		James Clerk Maxwell, “A dynamic theory of the electromagnetic field,” Royal Society Transactions, vol. 155, 1865, pp. 459-512.，本網站係以著作權法第 46、52、65 條合理使用本件作品。
26		CC BY SA	WIKIMEDIA COMMONS / File:Complex multi.svg / 2 August 2015 / IkamusumeFan， https://commons.wikimedia.org/wiki/File:Complex_multi.svg， CC BY-SA 4.0，2022/01/27 瀏覽。
26	× 1 i j k 1 1 i j k i i −1 k −j j j −k −1 i k k j −i −1	CC BY SA	WIKIPEDIA / Quaternion， https://en.wikipedia.org/wiki/Quaternion， CC BY-SA 3.0，2022/01/27 瀏覽。
26			WIKIMEDIA COMMONS / File:William Rowan Hamilton portrait oval combined.png / 30 August 2010, 11:19 (UTC)， https://commons.wikimedia.org/wiki/File:William_Rowan_Hamilton_portrait_oval_combined.png， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。

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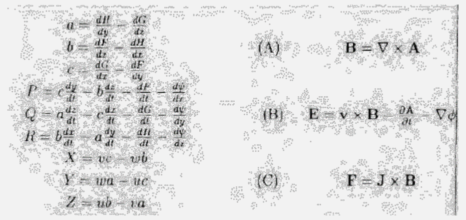
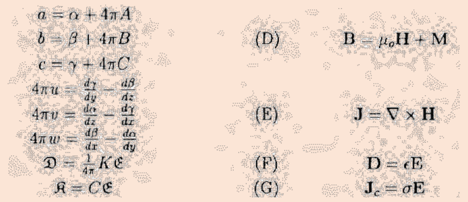
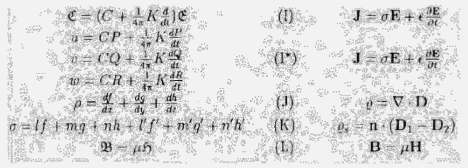
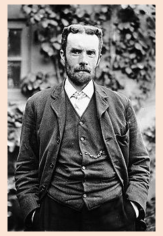
27			Tapan K. Sarkar, etc., History of Wireless, New Jerse1y: John Wiley & Sons, Inc., 2006, Fig. 3.11, p. 185. / James Clerk Maxwell, A Treaties on Electricity and Magnetism, Oxford: Clarendon Press, 1873, pp. 480-493.， 本網站係以著作權法第 46、52、65 條合理使用本件作品。
28			Tapan K. Sarkar, etc., History of Wireless, New Jerse1y: John Wiley & Sons, Inc., 2006, Fig. 3.11, p. 185. / James Clerk Maxwell, A Treaties on Electricity and Magnetism, Oxford: Clarendon Press, 1873, pp. 480-493.， 本網站係以著作權法第 46、52、65 條合理使用本件作品。
29			Tapan K. Sarkar, etc., History of Wireless, New Jerse1y: John Wiley & Sons, Inc., 2006, Fig. 3.11, p. 185. / James Clerk Maxwell, A Treaties on Electricity and Magnetism, Oxford: Clarendon Press, 1873, pp. 480-493.， 本網站係以著作權法第 46、52、65 條合理使用本件作品。
30			WIKIMEDIA COMMONS / File:Oheaviside.jpg / circa 1900 / Smithsonian Libraries， https://commons.wikimedia.org/wiki/File:Oheaviside.jpg， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。

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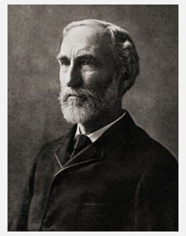
30			WIKIMEDIA COMMONS / File:Josiah Willard Gibbs -from MMS-.jpg / before 1903 . Uploaded in 2009 / Frontispiece of *The Scientific Papers of J. Willard Gibbs*, in two volumes, eds. H. A. Bumstead and R. G. Van Name, (London and New York: Longmans, Green, and Co., 1906)，https://commons.wikimedia.org/wiki/File:Josiah_Willard_Gibbs_-from_MMS-.jpg，本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
30	$\mathrm{AB} = AB\cos\theta$		Oliver Heaviside, Electromagnetic Theory, New York: Dover Publications, Inc., 1950, p. 38, ¶ 107.，本網站係以著作權法第 46、52、65 條合理使用本件作品。
30	$\mathrm{C} = V\mathrm{AB},\ C = AB\ sin\ \theta$		Oliver Heaviside, Electromagnetic Theory, New York: Dover Publications, Inc., 1950, p. 40, ¶ 111.，本網站係以著作權法第 46、52、65 條合理使用本件作品。
31	“used,…, to indicate the often-occurring operation of a line-integral in a closed circuit”		Oliver Heaviside, Electromagnetic Theory, New York: Dover Publications, Inc., 1950, p. 9, ¶ 33.，本網站係以著作權法第 46、52、65 條合理使用本件作品。

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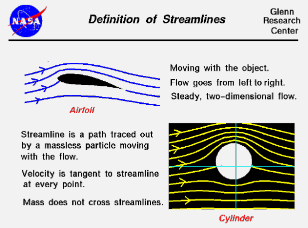
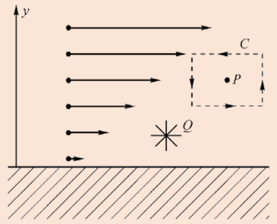
31	“The circuitation of the force is then called its ‘curl’.”		Oliver Heaviside, Electromagnetic Theory, New York: Dover Publications, Inc., 1950, p. 9, ¶ 36. ， 本網站係以著作權法第 46、52、65 條合理使用本件作品。
31	“The divergence of any flux is the amount of the flux leaving the unit volume.”		Oliver Heaviside, Electromagnetic Theory, New York: Dover Publications, Inc., 1950, p. 13, ¶ 51. ， 本網站係以著作權法第 46、52、65 條合理使用本件作品。
35			NASA， https://www.grc.nasa.gov/WWW/k-12/VirtualAero/BottleRocket/airplane/stream.html， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
36			鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.， 本網站係以著作權法第 46、52、65 條合理使用本件作品。

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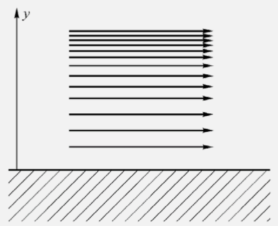
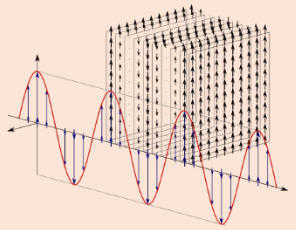
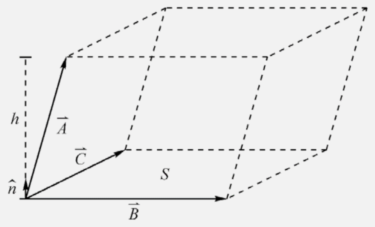
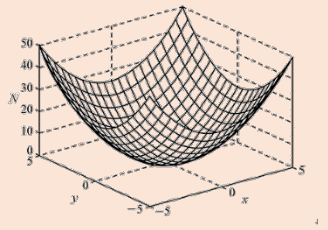
36			鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.， 本網站係以著作權法第 46、52、65 條合理使用本件作品。
36			WIKIMEDIA COMMONS / File:Linear Polarization Linearly Polarized Light plane wave.svg / Dave3457， https://commons.wikimedia.org/wiki/File:Linear_Polarization_Linearly_Polarized_Light_plane_wave.svg， Public domain，2022/01/27 瀏覽。
37			鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.， 本網站係以著作權法第 46、52、65 條合理使用本件作品。
38			鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.， 本網站係以著作權法第 46、52、65 條合理使用本件作品。

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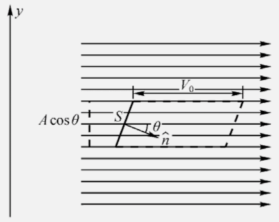
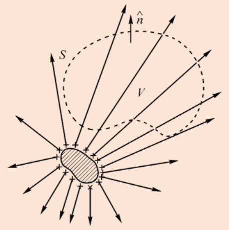
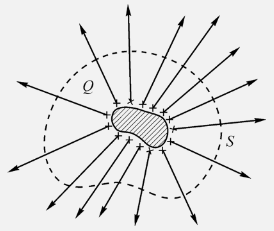
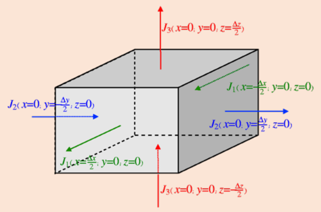
39			鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.， 本網站係以著作權法第 46、52、65 條合理使用本件作品。
40			鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.， 本網站係以著作權法第 46、52、65 條合理使用本件作品。
40			鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.， 本網站係以著作權法第 46、52、65 條合理使用本件作品。
41、54			Massachusetts Institute of Technology / The Divergence Theorem， http://pruffle.mit.edu/3.016-2005/Lecture_16_web/node2.html， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。

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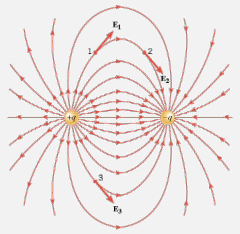
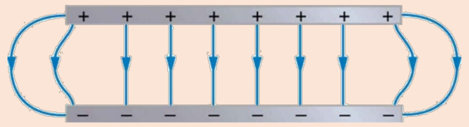
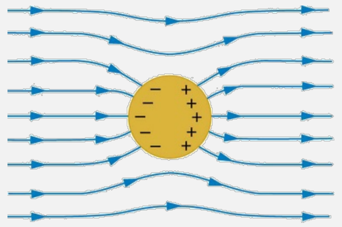
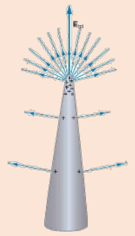
43			WebAssign/Cengage / The electric field lines of an electric dipole are curved and extend from the positive to the negative charge. At any point, such as 1, 2, or 3, the field created by the dipole is tangent to the line through the point.，https://demo.webassign.net/ebooks/cj6demo/pc/c18/read/main/c18x18_7_ref.htm，本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
43		CC BY	Lumen Learning / Physics / Conductors and Electric Fields in Static Equilibrium，https://courses.lumenlearning.com/physics/chapter/18-7-conductors-and-electric-fields-in-static-equilibrium/，CC BY 4.0，2022/01/27 瀏覽。
43		CC BY	Lumen Learning / Physics / Conductors and Electric Fields in Static Equilibrium，https://courses.lumenlearning.com/physics/chapter/18-7-conductors-and-electric-fields-in-static-equilibrium/，CC BY 4.0，2022/01/27 瀏覽。
43		CC BY	Lumen Learning / Physics / Conductors and Electric Fields in Static Equilibrium，https://courses.lumenlearning.com/physics/chapter/18-7-conductors-and-electric-fields-in-static-equilibrium/，CC BY 4.0，2022/01/27 瀏覽。

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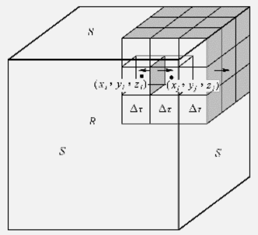
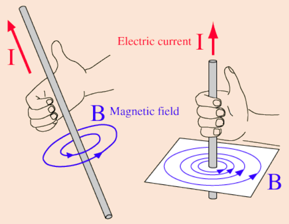
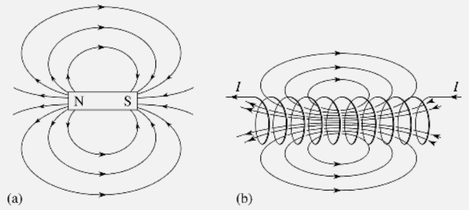
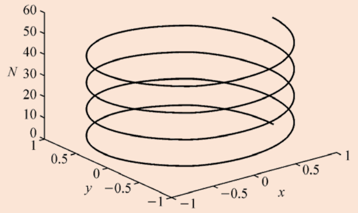
44			鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.， 本網站係以著作權法第 46、52、65 條合理使用本件作品。
45			WIKIMEDIA COMMONS / File:V-1 right hand thumb rule.gif / Oguraclutch， https://commons.wikimedia.org/wiki/File:V-1_right_hand_thumb_rule.gif， Public domain，2022/01/27 瀏覽。
45			CBSE SCIENCE / Learning Resource for CBSE students， https://cbsescience.in/2017/09/27/magnetic-field-through-a-current-carrying-conductor/， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
47			鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.， 本網站係以著作權法第 46、52、65 條合理使用本件作品。

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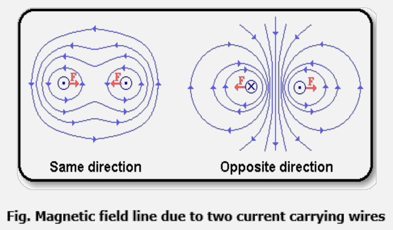
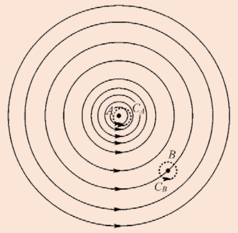
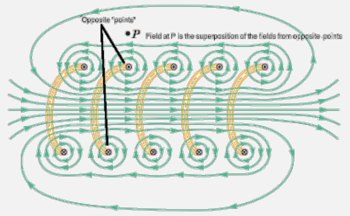
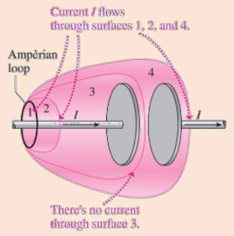
53			goprep / What is the magne， https://goprep.co/what-is-the-magnetic-field-midway-between-two-parallel-i-1nlo17， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
53			鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.， 本網站係以著作權法第 46、52、65 條合理使用本件作品。
53			Stock Exchange / What are the limits of validity for the magnetic field of a solenoid?， https://physics.stackexchange.com/questions/102312/what-are-the-limits-of-validity-for-the-magnetic-field-of-a-solenoid，本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。
55-56			Physics Department, University of Louisville / Displacement Current， https://www.physics.louisville.edu/cldavis/phys299/notes/mag_displacement.html， 本網站係以著作權法第 46、52、65 條合理使用本件作品，2022/01/27 瀏覽。

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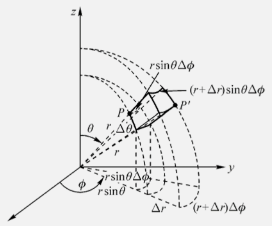
57			鄭士康, 電磁波, 台北市: 全華, 2006, 附錄 B.， 本網站係以著作權法第 46、52、65 條合理使用本件作品。